

Math 2090 (McGehee) Problem Set 8, Due Wednesday, December 1, 2004
Please fold papers the long way, put name on outside.

A. Solve the initial value problem

$$y'' + 6y' + 9y = 0 \quad y(0) = 2, \quad y'(0) = -5.$$

Show your procedure in detail. You may use any method you like.

B. Using the method of variation of parameters, find the general solution of the differential equation

$$y'' + 6y' + 9y = \frac{2e^{-3x}}{x^2 + 1}.$$

Show your procedure in detail.

C. Find the general solution of this differential equation of Cauchy-Euler type:

$$x^2y'' - xy' - 35y = 0 \quad (x > 0).$$

Show your procedure clearly and completely.

D. Find the general solution of this differential equation of Cauchy-Euler type:

$$x^2y'' - 4xy' + 6y = x^4 \sin x \quad (x > 0).$$

Show your procedures clearly and completely.

E. Find the general solution for this system:

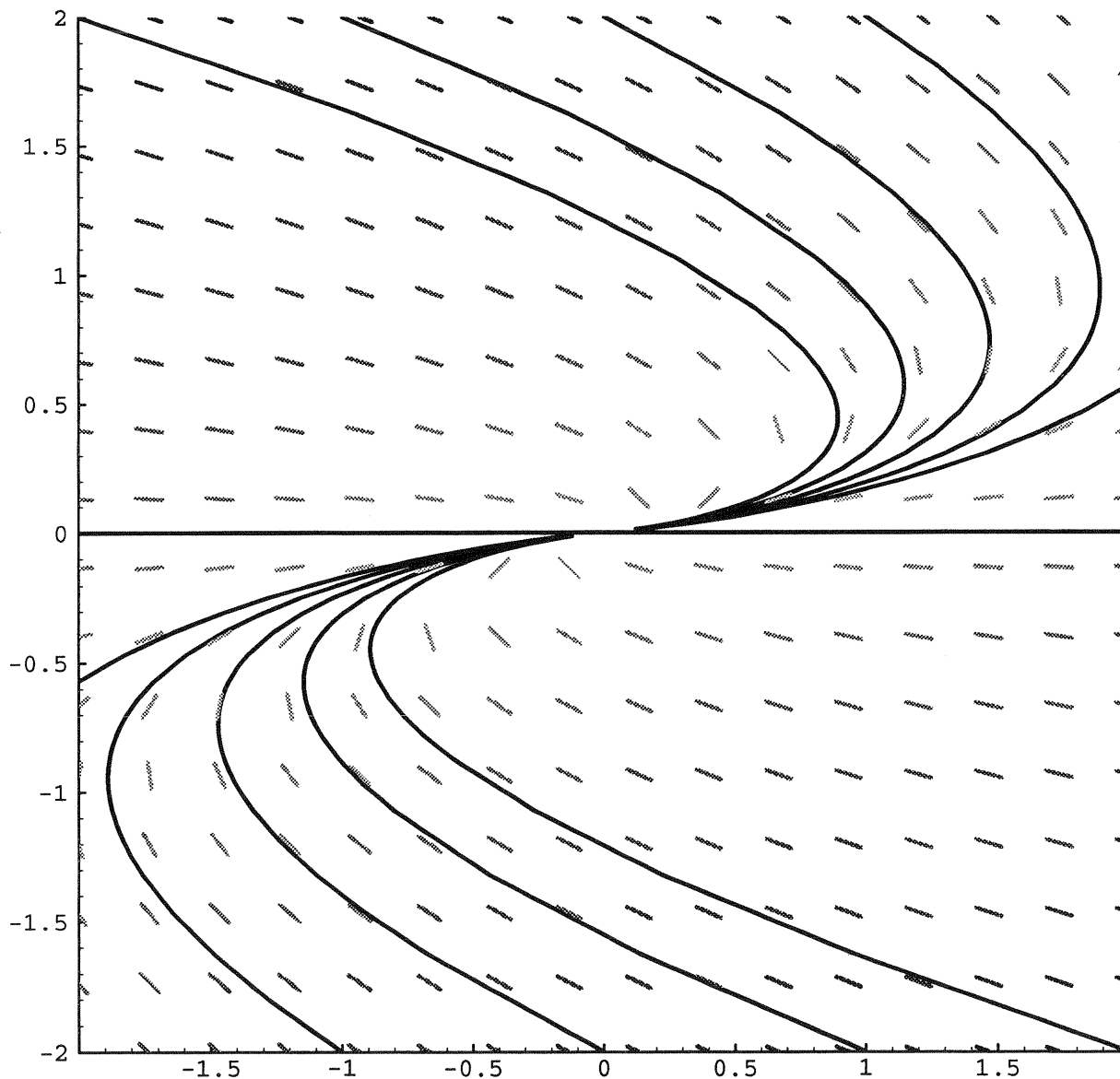
$$x' = x - y, \quad y' = -2y.$$

Sketch the orbital portrait as accurately as you can, using computer software if you like. Draw arrowheads on several of the orbits to show which way $(x(t), y(t))$ is moving as t increases.

■ You are given the general solution of a system, and you are given the orbital portrait. Draw arrowheads on all the orbits shown in the portrait to show which way points move along those orbits. Also, determine and write down the system of differential equations of which this is the solution. Show how you determine your answer.

$$x = c_1 e^{-t} + 2 c_2 t e^{-t}$$

$$y = c_2 t e^{-t}$$



G (Triple Credit)

Below you will find four systems of equations $x' = Ax$, where A is 2×2 . They are labeled P, Q, R, and S. You will also find a number of orbital portraits, labeled I, II, III, and so forth. In each blank, write the numeral of the correct orbital portrait:

Orbital portrait _____ is for system P.

Orbital portrait _____ is for system Q.

Orbital portrait _____ is for system R.

Orbital portrait _____ is for system S.

System P: $x' = -x + 2y, y' = -y$

System Q: $x' = x, y' = x - y$

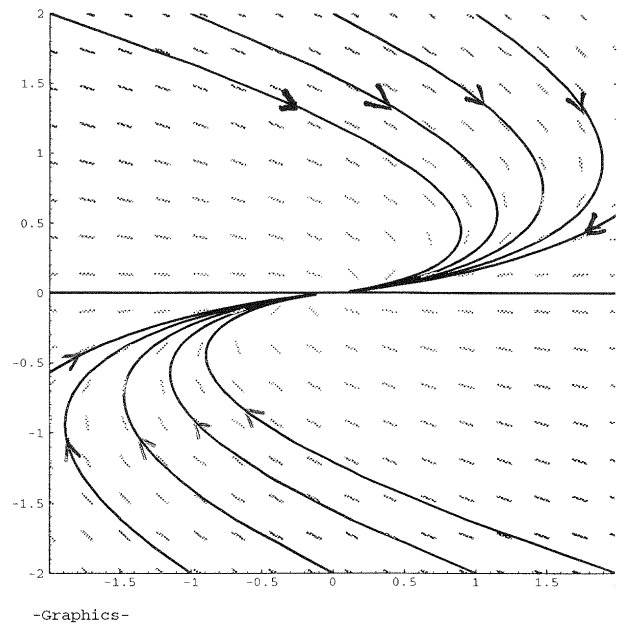
System R: $x' = -x + 4y, y' = -3x - 2y$

System S: $x' = -x - y, y' = -2y$

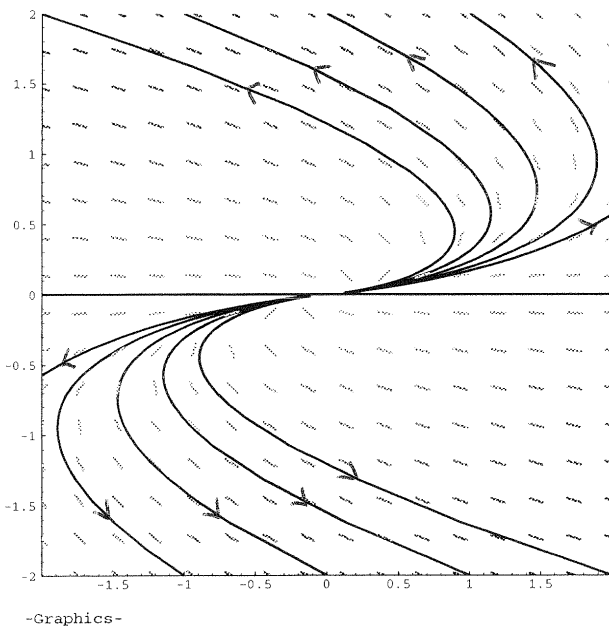


_____, present adequate justification for each of your four answers above.

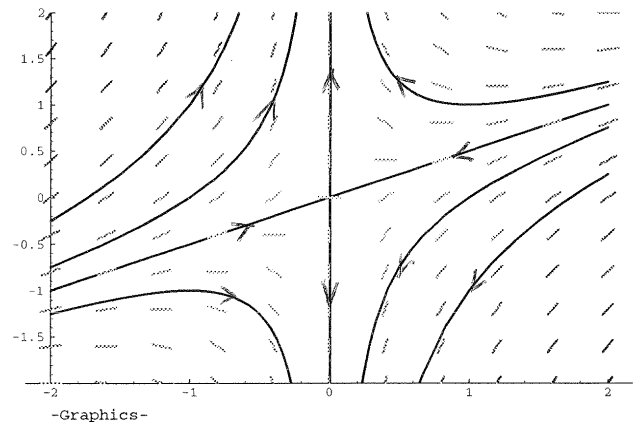
Orbital Portrait I



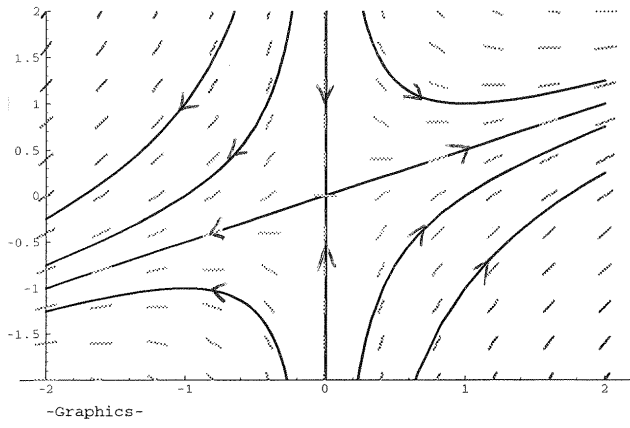
Orbital Portrait II



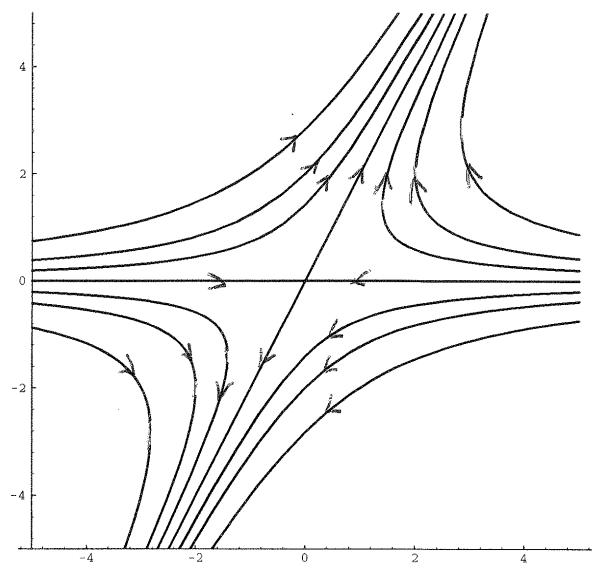
Orbital Portrait III



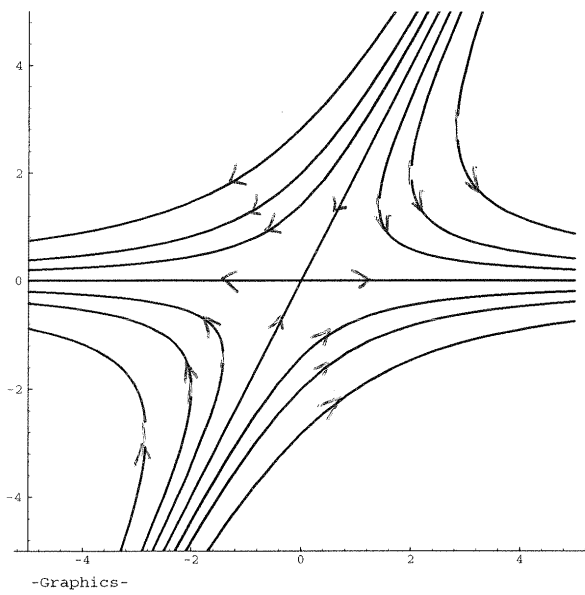
Orbital Portrait IV



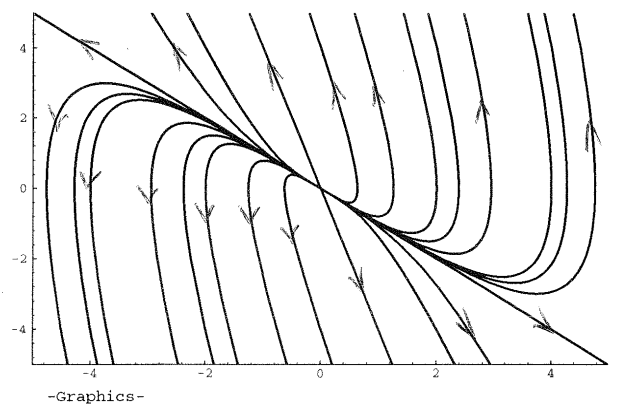
Orbital Portrait V



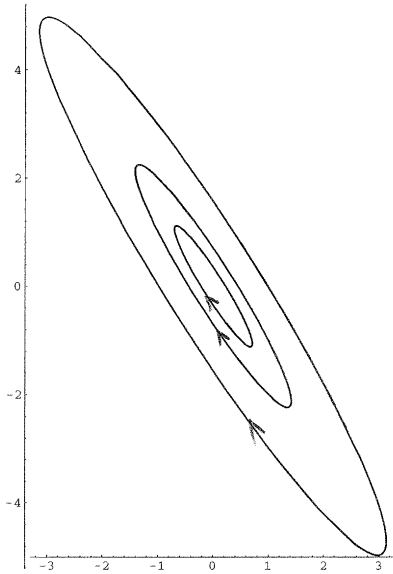
Orbital Portrait VI



Orbital Portrait VII

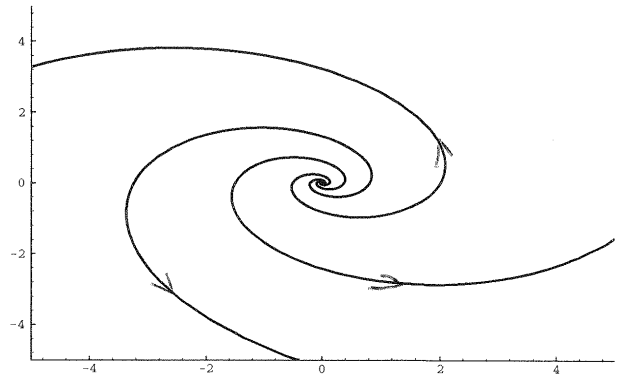


Orbital Portrait VIII



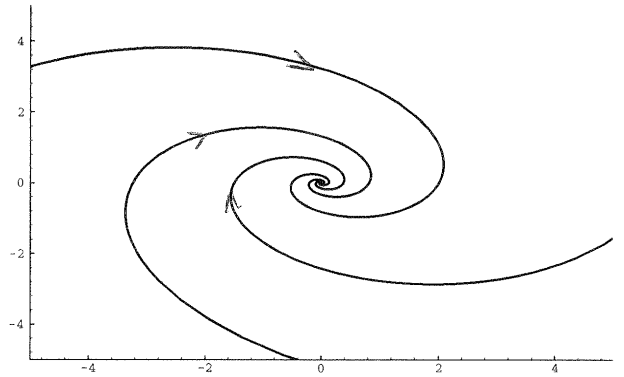
-Graphics-

Orbital Portrait IX



-Graphics-

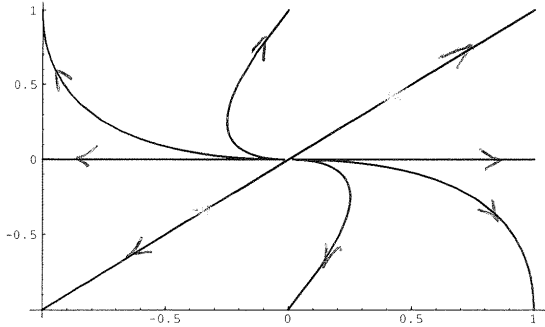
Orbital Portrait X



-Graphics-

Orbital Portrait XI

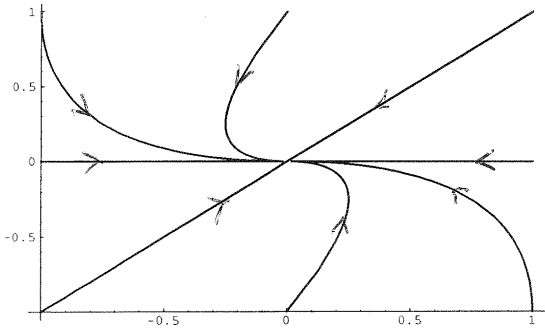
~~Asymptotically stable, Improper node~~



-Graphics-

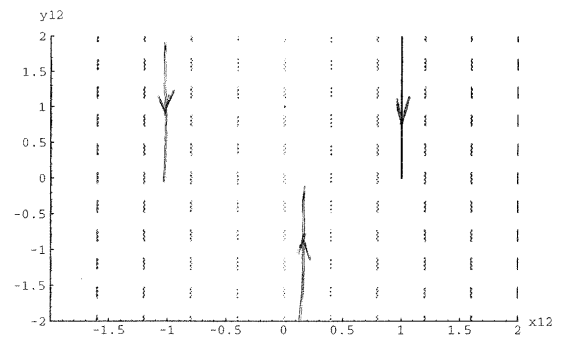
Orbital Portrait XII

Asymptotically stable, Improper node



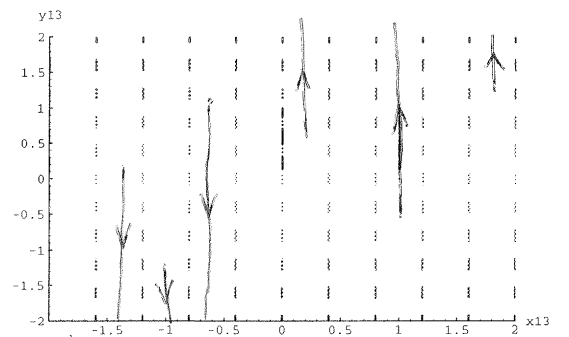
-Graphics-

Orbital Portrait XIII



-Graphics-

Orbital Portrait XIV



-Graphics-