Mathematics 2090 (McGehee) Test 1 September 29, 2004

1. At the end of the test, you will find a picture of the direction field for the DE

$$y' = \frac{1}{2}(1 - \frac{y}{3})y.$$

Just by looking at the picture, find the answers to the following questions.

a. Find the solution $y(x)$ of the given DE such that $y(0) = 3$.

b. If $y(x)$ is the solution of the given DE such that $y(0) = 1$, what is $\lim_{x\to\infty} y(x)$?

2. In solving certain linear constant-coefficient nonhomogeneous differential equations, one may use the method of undetermined coefficients to find a particular solution after finding the homogeneous solution. The method is to recognize that there will be a solution of a certain form, such that all that remains is to determine one or more constants. For example, for the DE

$$y'' + 8y' + 15y = \sin 2x,$$

the homogeneous solution consists of all linear combinations of e^{-5x} and e^{-3x}, and we can then see that there must be a particular solution of the form $A\cos 2x + B\sin 2x$. In each of the following cases, just write down the form of a particular solution. Do not evaluate the undetermined coefficients. The answer to item z is provided, to make sure you understand what is required.

z. (Sample question) $y'' + 8y' + 15y = e^{-5x}$. Answer: Axe^{-5x}

a. $y'' + 8y' + 15y = \sin 3x$.

b. $y'' + 4y = \sin 2x$.

c. $y'' + 8y' + 15y = e^{x}$.

3. Find the general solution of this DE:

$$\frac{1}{2}x''(t) + \frac{1}{10}x'(t) + \frac{1}{100}x(t) = 0.$$

4. Let $f_1(x) = \sin 2x$, $f_2(x) = \cos 2x$. Compute the Wronskian $W[f_1, f_2](x)$. One can determine whether functions are independent by looking at their Wronskian. Are these two functions independent?

5. Find the unique solution of this initial value problem, and sketch its graph:

$$y'' + 3y' + 2y = 0, \quad y(0) = 1, \quad y'(0) = -3.$$

This problem describes a mass-spring system. Is the system underdamped or overdamped? Sketch the graph of the motion. Does $y(t) = 0$ for some $t > 0$? If so, what is the value of t?

6. As accurately as you can, sketch the direction field for the DE $y' = 2xy^2$ in the region $0 < x < 2$, $0 < y < 2$. Then find the solution of this initial value problem:

$$y' = 2xy^2, \qquad y(1) = 1.$$

7. Find the solution of this first-order equation with initial value: :

$$\frac{dy}{dx} + \frac{1}{2}\, y = 2 + x, \qquad y(0) = 2.$$

8. Suppose that g is a continuous function. Explain briefly—going as far as you can without knowing more about g—how you would proceed to solve the following problem by the method of variation of parameters:

$$y'' + y = g(x).$$

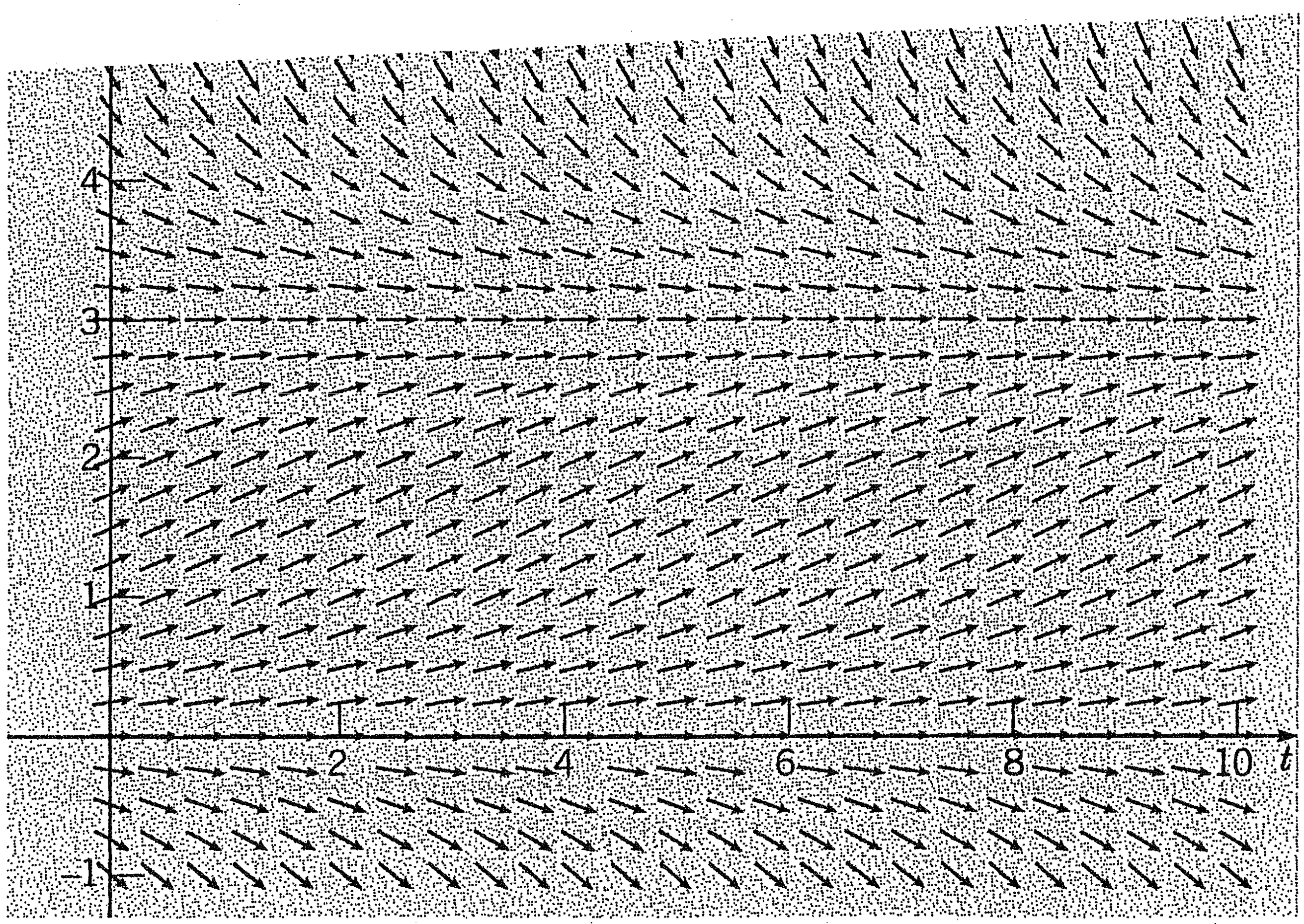

FIGURE 2.5.2 Direction field for $dy/dt = r(1 - y/K)y$ with $r = 1/2$ and $K = 3$.