Math 2090 (McGehee) Test One Notes October 4, 2004

1a the constant function $y(x) \equiv 3$.

b. If $y(0) = 1$, $\lim_{x\to\infty} y(x) = 3$.

2. a $A\cos 3x + B\sin 3x$

b. $Ax\cos 3x + Bx\sin 3x$

c. Ae^x.

3. $\frac{1}{2}x'' + \frac{1}{10}x' + \frac{1}{100}x = 0$. Auxiliary equation:

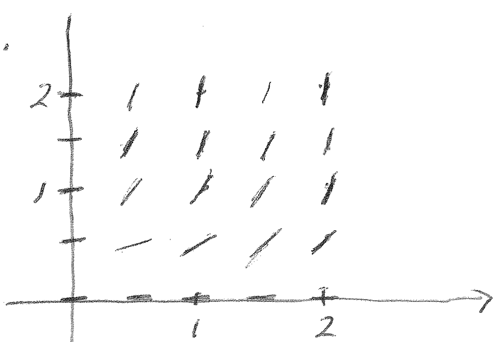
$\frac{1}{2}r^2 + \frac{1}{10}r + \frac{1}{100} = 0$. Roots: $\dfrac{-\frac{1}{10} \pm \sqrt{\frac{1}{100} - \frac{2}{100}}}{1}$

$= -\frac{1}{10} \pm \frac{i}{10}$. $y = c_1 e^{-t/10}\cos\frac{t}{10} + c_2 e^{-t/10}\sin\frac{t}{10}$.

4. $W[f_1, f_2](x) = \det\begin{pmatrix} \sin 2x & \cos 2x \\ 2\cos 2x & -2\sin 2x \end{pmatrix} \equiv -2 \neq 0$,

so f_1 and f_2 are independent.

6.



$\frac{dy}{dx} = 2xy^2$. $\int_1^Y \frac{dy}{y^2} = 2\int_1^X x\,dx$.

$-\frac{1}{y}\Big|_1^Y = 2\frac{x^2}{2}\Big|_1^X$.

$-\frac{1}{Y} + 1 = X^2 - 1$. So $\underline{Y = \frac{1}{2 - X^2}}$

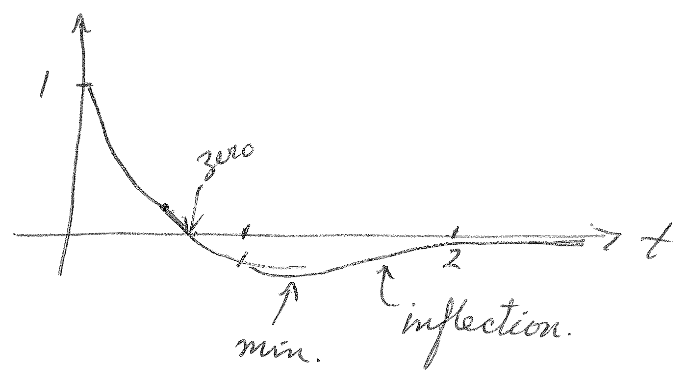
5. General solution: $y = C_1 e^{-2t} + C_2 e^{-t}$.

Then $y' = -2C_1 - C_2 e^{-t}$.

$$\left.\begin{aligned} y(0) &= C_1 + C_2 = 1 \\ y'(0) &= -2C_1 - C_2 = -3 \end{aligned}\right\} \Rightarrow C_1 = 2,\ C_2 = -1$$

$y = 2e^{-2t} - e^{-t} = e^{-t}(2e^{-t} - 1) = 0$ when

$e^{-t} = \frac{1}{2}$ or $e^{t} = 2$, which happens only when $t = \ln 2$.



overdamped.

Solution of 2b (not asked for on the test).

$y'' + 4y = \sin 2x$. $\quad y_h = C_1 \cos 2x + C_2 \sin 2x$

Try $y = Ax\cos 2x + Bx \sin 2x$.

Then $y' = +2Bx\cos 2x - 2Ax\sin 2x + A\cos 2x + B\sin 2x$

and $y'' = -4Ax\cos 2x - 4Bx\sin 2x + 2B\cos 2x - 2A\sin 2x$

So $y'' + 4y = 2B\cos 2x - 2A\sin 2x$;

we want this to equal $\sin 2x$.

So $B = 0$, $A = -\frac{1}{2}$ works.

$y = C_1 \cos 2x + C_2 \sin 2x - \frac{1}{2} x \cos x$.

7. $y' + \frac{1}{2}y = 2+x$. <u>Method 1</u>: The complementary solution is $Ce^{-\frac{1}{2}x}$. To find a particular solution, use Undetermined Coefficients: If $y = Ax+B$, then $y' + \frac{1}{2}y = (A + \frac{1}{2}B) + \frac{1}{2}Ax \equiv 2+x$, so $A=2$, $B=0$. General solution: $y = 2x + Ce^{-x/2}$. If $y(0)=2$, then $C=2$. $\underline{y = 2x + 2e^{-x/2}}$.

<u>Method 2</u>: First-order linear. Use the integrating factor $e^{x/2}$: $e^{x/2}y' + \frac{1}{2}e^{x/2}y = (e^{x/2}y)' = 2e^{x/2} + xe^{x/2}$. So $e^{x/2}y = 2xe^{x/2} + C$ (integration by parts), so $y = 2x + Ce^{-x/2}$ ... and $C=2$.

8. The complementary solution of $y'' + y = g$ is $C_1\cos x + C_2 \sin x$. A particular solution will be $u_1\cos x + u_2 \sin x$ where

$$\begin{cases} u_1'\cos x + u_2'\sin x = 0 \\ u_1'(-\sin x) + u_2'\cos x = g(x) \end{cases}$$

Thus $u_1' = -g(x)\sin x$, $u_2' = +g(x)\cos x$, and we obtain u_1 and u_2 by integrating.

$$y = -\cos x \int g(x)\sin x\,dx + \sin x \int g(x)\cos x\,dx.$$