Notes on Test 3, Math 2090 (McGehee) 11/17/04

1a e^{at} because $\int_0^\infty e^{at}e^{-st}\,dt = \frac{1}{a-s}e^{-(s-a)t}\Big|_0^\infty = \frac{1}{s-a}$ $(s>a)$

b. $\frac{n!}{s^{n+1}}$ because (using #11 of the table) $t^n e^{at} \xrightarrow{\mathcal{L}} \frac{n!}{(s-a)^{n+1}}$

(apply with $a=0$)

c. $\frac{e^{-cs}}{s}$ because $\int_0^\infty e^{-st}u_c(t)\,dt = \int_c^\infty e^{-st}\,dt = \frac{e^{-st}}{-s}\Big|_c^\infty = \frac{e^{-sc}}{s}$

d. $e^{+ct}f(t)$ because $\int_0^\infty e^{-st}e^{ct}f(t)\,dt = \int_0^\infty e^{-(s-c)t}f(t)\,dt$

$= F(s-c)$.

2. (1) $\frac{s+3}{s^2+9}$ (2) $\frac{1}{s+2}$ (3) $\frac{2}{(s-5)^3}$ (4) $\frac{e^{-\pi s}}{(s+1)^2}$

(5) $\mathcal{L}\{\sin t\} = \frac{1}{s^2+1}$, so $\mathcal{L}\{t\sin t\} = -\frac{d}{ds}\frac{1}{s^2+1} = \frac{+2s}{(s^2+1)^2}$.

Note: The convolution is not involved in (5).

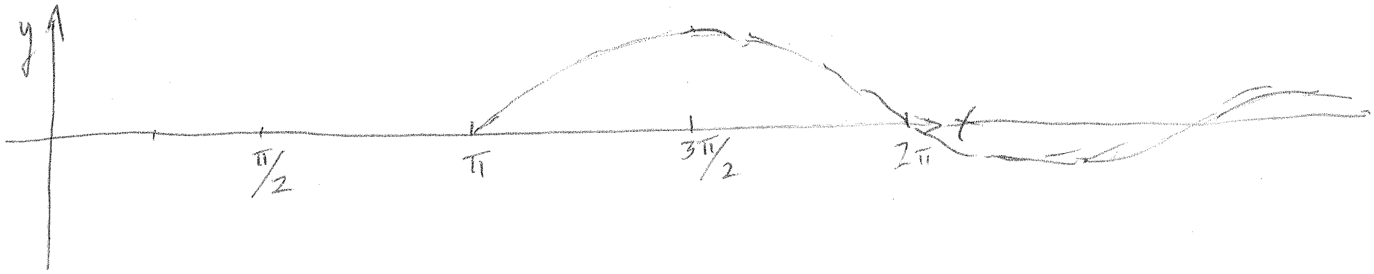
$\left(\mathcal{L}^{-1}\left\{\frac{1}{s^2(s^2+1)}\right\}\right.$ would be $\left.\int_0^t (t-v)\sin v\,dv\right)$

3. a. e^{-t} b. te^{-t} c. $\frac{6(s+1)}{(s^2+2s+1)+5} = \frac{6(s+1)}{(s+1)^2+5}$

$\xrightarrow{\mathcal{L}^{-1}} 6e^{-t}\cos\sqrt{5}\,t$.

4. $s^2Y(s) + 2sY(s) + 2Y(s) = e^{-\pi s}$, $Y(s) = \frac{e^{-\pi s}}{(s+1)^2+1}$,

$y(t) = u_\pi(t)\,e^{-(t-\pi)}\sin(t-\pi)$



Math 2090 (McGehee) Test 3 November 17, 2004

This is a closed-book, closed-notes test.

Computers and calculators are not allowed.

Make your procedures clear.

A partial table of Laplace transforms is provided for your use.

1. There are four blanks in the table of Laplace transforms. Fill them in correctly.

2. In each case, find the Laplace transform of the given function of t.

   (1) $\cos 3t + \sin 3t$

   (2) e^{-2t}

   (3) $t^2 e^{5t}$

   (4) $u_\pi(t)(t-\pi)e^{-(t-\pi)}$

   (5) $t \sin t$

3. In each case, find the inverse Laplace transform of the given function of s.

   a. $\dfrac{1}{s+1}$

   b. $\dfrac{1}{(s+1)^2}$

   c. $\dfrac{6(s+1)}{s^2+2s+6}$

4. Show in detail how to solve this initial value problem using the method of Laplace transforms, and sketch the graph of the solution:

$$y'' + 2y' + 2y = \delta(t-\pi), \quad y(0) = 0, \quad y'(0) = 0.$$

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	
1. 1	$\dfrac{1}{s}, \quad s > 0$	
2.	$\dfrac{1}{s-a}, \quad s > a$	a
3. t^n; $n =$ positive integer		b
4. t^p, $p > -1$	$\dfrac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$	
5. $\sin at$	$\dfrac{a}{s^2+a^2}, \quad s > 0$	
6. $\cos at$	$\dfrac{s}{s^2+a^2}, \quad s > 0$	
7. $\sinh at$	$\dfrac{a}{s^2-a^2}, \quad s > \lvert a \rvert$	
8. $\cosh at$	$\dfrac{s}{s^2-a^2}, \quad s > \lvert a \rvert$	
9. $e^{at} \sin bt$	$\dfrac{b}{(s-a)^2+b^2}, \quad s > a$	
10. $e^{at} \cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}, \quad s > a$	
11. $t^n e^{at}$, $n =$ positive integer	$\dfrac{n!}{(s-a)^{n+1}}, \quad s > a$	
12. $u_c(t)$		c
13. $u_c(t) f(t-c)$	$e^{-cs} F(s)$	
14.	$F(s-c)$	d
15. $f(ct)$	$\dfrac{1}{c} F\left(\dfrac{s}{c}\right), \quad c > 0$	
16. $\int_0^t f(t-\tau) g(\tau)\, d\tau$	$F(s)G(s)$	
17. $\delta(t-c)$	e^{-cs}	
18. $f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - \cdots - f^{(n-1)}(0)$	
19. $(-t)^n f(t)$	$F^{(n)}(s)$	

Table from the book by Boyce + DiPrima