AMS SECTIONAL MEETING, BATON ROUGE, MARCH 14–16, 2003

Frames, Wavelets, and Tomography

Meeting # 984, AMS Code AMS SS E1

Speakers, Titles, and Abstracts

(1) Baggett, Lawrence (baggett@euclid.colorado.edu)
Title: Generalized Multiresolution Analysis, a New Look.
Abstract # : 984-43-85
Abstract: An updated and modernized notion of a generalized multiresolution analysis in a Hilbert space is presented. Groups (possibly uncountable) of analyzing operators, for example other than $\mathbb{Z}^n$, are allowed, and connections are made between such multiresolution structures and possibly uncountable "Plancherel wavelet analysis."

(2) Balan, Radu (radu.balan@scr.siemens.com)
Title: A frame-Riesz basic sequence duality result for wavelet sets.
Abstract # : 984-40-25
Abstract: For Gabor sets, $(g; a, b)$, it is known that $(g; a, b)$ is a frame if and only if $(g; 1/b, 1/a)$ is a Riesz basis for its span. In particular, for every g there is a_0 such that for every $a < a_0$, there is a $b_m = b_m(a) > 0$ so that for every $b < b_m$, $(g; a, b)$ is a frame, and $(g; 1/b, 1/a)$ is a Riesz basis sequence. In this talk we shall consider a similar problem for wavelet sets $(\Psi; a, b)$. The main result reads as follows. Given an admissible Ψ, there is $a_0 > 0$ so that for every $a < a_0$, there is a $b_m = b_m(a)$ so that for every $b < b_m$, $(\Psi; a, b)$ is a frame, and $(\Psi; 1/b, 1/a)$ is a Riesz basis for its span.

(3) Boncek, John (jboncek@pegasus.cc.ucf.edu)
Title: Generalization of Frame Potentials
Abstract # : 984-43-222
Abstract: The concept of frame potentials has recently emerged as a useful technique for discovering and analyzing tight frames. In our work, we introduce variations on this definition and study the corresponding frame behaviors.

(4) Bownik, Marcin
Title: Affine Frames, GMRA's, and the Canonical Dual.
Joint with Eric Weber
Abstract # : 984-42-152
Abstract: We show that if the canonical dual of an affine frame has the affine structure, with the same number of generators, then the core subspace V_0 is shift invariant. We demonstrate, however, that the converse is not true. We apply our results in the setting of oversampling affine frames, as well as computing the period of a Riesz wavelet, which answers in the affirmative a conjecture of Daubechies and Han. Additionally, we completely characterize when the canonical dual of a quasi-affine frame has the quasi-affine structure.

(5) Casazza, Pete (pete@math.missouri.edu)
Title: A physical interpretation for finite tight frames.
Joint with: M. Fickus, fickus@math.cornell.edu, Jelena Kovacevic, j.kovacevic@ieee.org, Manuel T. Leon, mleon@math.missouri.edu, Janet Crandell Tremain, j.tremain@math.missouri.edu
Abstract # : 984-15-32
Abstract: We discuss some results which give a physical interpretation for finite tight frames. Building on the concept of a *frame force* (which leads naturally to a *frame potential* introduced by Benedetto and Fickus, we show how the theory of frames is connected to the application of several standard notions of classical mechanics to the frame force. This allows us to use "physical rationale" to anticipate new results in frame theory. Armed with the *Physics* of tight frames, we are lead to a *fundamental inequality* that all tight frames must satisfy. Moreover, given a sequence of numbers $\{a_n\}$ satisfying the fundamental inequality, there must exists a tight frame $\{\varphi_n\}$ with $\|\varphi_n\| = a_n$, for all n. As a consequence, we discover that tight frames can be custom build for most applications as long as the fundamental inequality is not violated and the requirements for the application are not too "rigid". Finally, it it is necessary to violate the fundamental inequality for a particular application, we will show the structure of the available frames which are the closest to being tight (in the sense of minimizing potential energy).

(6) Dutkay, Dorin (ddutkay@math.uiowa.edu)
Title: Wavelet representations and the Ruelle transfer operator
Abstract # : 984-43-127
Abstract: We introduce an abstract version of multiresolution analysis which stresses the connection with the spectral properties of the Ruelle transfer operator. Using this theory we were able to give a complete picture of the spectrum of the Ruelle operator and dilate normalized tight frame MRA-wavelets to orthogonal wavelets in a bigger Hilbert space. In this way a large class of super-wavelets is obtained

(7) Han, Deguan (dhan@pegasus.cc.ucf.edu)
Title: Characterization of Functional Gabor Frame Multipliers.
Abstract # : 984-42-62
Abstract: We will present a simple characterization of functional Gabor frame multipliers.

(8) Johnson, Brody (brody@math.wustl.edu)
Title: Co-affine systems in $\mathbb{R}^d$.
Abstract # : 984-42-256
Abstract: We extend the proof of the non-existence of co-affine frames for L^2 from the one-dimensional case to that of expanding dilations in $\mathbb{R}^d$. We then consider the problem of identifying subspaces on which co-affine systems may admit frame-like inequalities. In the MRA setting we show that frame-like inequalities do hold on certain fundamental subspaces of the MRA. We then exhibit necessary conditions for a general co-affine system to admit frame-like inequalities on band-limited subspaces. In the case of the Shannon wavelet these results dictate that Parseval's identity holds on the band-limited subspace having bandwidth 2, but can not hold with any bandwidth larger than 2.

(9) Jorgensen, Palle (pjorgen@blue.weeg.uiowa.edu)
Title: Orthogonality relations for wavelets and for fractals.
Abstract # : 984-42-72
Abstract:

(10) Kaiser, Gerald (kaiser@wavelets.com)

Title: Physical wavelets and their extended-delta source distributions.

Abstract # : 984-78-34

Abstract: Physical wavelets W_z are, roughly, causal acoustic or electromagnetic waves emitted by a "source point" $z = x + iy$ located in complex spacetime. We clarify this statement in terms of hyperfunctions and show that the source of W_z is (essentially) a compactly supported distribution in real spacetime centered at x, with the vector y (which must belong to the future cone) giving its extension about x. The space components of y give the orientation and radius of a disk from which W_z is launched, and its time component gives the duration. Consequently, W_z is a pulsed bean that can be directed and focused as sharply as desired by choosing y. These are multidimensional versions of the usual parametrization of $1D$ wavelets by time and scale. The associated resolutions of unity provide a natural method to decompose and reconstruct acoustic and electromagnetic waves in terms of pulsed-beam wavelets.

(11) Kornelson, Keril (keri@math.tamu.edu)

Title: Ellipsoidal Tight Frames in Finite and Infinite Dimensional Hilbert Spaces.

Joint with: Ken Dykema, kdykema@math.tamu.edu, Dan Freeman, dfreeman@guilford.edu, David Larson, larson@math.tamu.edu, Marc Ordower, mordower@rmwc.edu, Eric Weber, ESW@uwyo.edu

Abstract # : 984-47-105

Abstract: Tight frames on a Hilbert space are useful in a variety of signal processing applications, but implementing them may require the frame to have additional properties. Because many aspects of frame theory are geometric in nature, a natural question which arises is which surfaces within a Hilbert space contain tight frames. We demonstrate that any arbitrary ellipsoid will contain a tight frame, and furthermore, will contain a tight frame of any given cardinality at least as big as the dimension of the space.

(12) Kuchment, Peter (kuchment@math.tamu.edu)

Title: Complex analysis in some tomographic problems.

Joint with Leonid Kunyansky, leonk@math.arizona.edu

Abstract # : 984-44-138

Abstract: Analysis in several complex variables has proven to be one of the important analytic techniques for the computerized tomography. Recent developments of this kind include in particular the 180 degrees acquisition in 2D and rotating slant-hole 3D single-photon emission tomography. The talk will address some of these problems.

(13) Lammers, Mark (Mark.Lammers@wwu.edu)

Title: Bracket Products for Weyl-Heisenberg (Gabor) Frames

Joint work with Pete G Casazza

Abstract # : 984-46-258

Abstract: We give an overview of the L^1 function-valued inner product on $L^2(\mathbb{R})$ known as the bracket product. We show that this inner product has a Bessel's inequality, a Riesz Representation Theorem, and a Gram–Schmidt process. We then apply this to Weyl–Heisenberg frames to show that there exist "compressed" versions of the operators associated with Weyl-Heisenberg systems. Finally, we consider this bracket product on Wiener Amalgam spaces to produce a Hilbert C^* Module and an analogue of the Wiener Algebra related to Weyl-Heisenberg Systems.

(14) Ludu, Andy (ludua@nsula.edu)

Title: Wavelet analysis of soliton solutions of nonlinear partial differential equations.

Abstract # : 984-43-198

Abstract: A multi-scale wavelet analysis is used in order to obtain qualitative properties of soliton-like solutions for nonlinear partial differential equations (NLPD). The procedure provides relations between the velocity and the geometry of traveling localized solutions of NLPD whose exact solutions are difficult/impossible to be obtained. The approcah consists in substitution of all terms of the NPDE with corresponding average values of the solution and its derivatives. We are using (but we are not limited to) Gaussian filtering and Morlet continuous wavelets in order to expand the soliton-like solutions around their maxima. Around such points the traveling solutions can be described by a finite number of scales related to the dominant terms. For NLPDE with known soliton solutions comparison between the exact solutions and this approach gives very good results. The approach, though simple, provides good power of prediction.

(15) Massopust, Peter (pmassopust@varco.com)

Title: Multiscale Structures and Coxeter Groups

Abstract # : 984-42-221

Abstract: Abstract: We show how elements from the theory of Coxeter groups and foldable figures may be employed to obtain special multiscale structures. These multiscale structures give rise to scaling vectors and multiwavelets that are defined on the fundamental domains of these group objects. We show how elements from the theory of Coxeter groups and foldable figures may be employed to obtain special multiscale structures. These multiscale structures give rise to scaling vectors and multiwavelets that are defined on the fundamental domains of these group objects.

(16) Papadakis, Manos (mpapadak@math.uh.edu)

Title: Non-separable Radial Frame Multiresolution Analysis in Multidimensions.

Joint with: G Gogoshin, ggogoshin@uh.edu, I A Kakadiaris, ioannisk@uh.edu, D J Kouri, kouri@uh.edu, D K Hoffman, hoffman@ameslab.gov

Abstract # : 984-42-76

Abstract: In this paper we present a non separable multiresolution structure based on frames which is defined by radial scaling functions,which are minimally supported in the frequency. We also construct the resulting frame multiwavelets, which can be isotropic as well. Our construction can be carried out in any number of dimensions and for a big variety of dilation matrices.

(17) Parker, Judith (Judith.Jesudason@colorado.edu)

Title: Operators associated to Generalized Filter Functions

Abstract # : 984-42-100

Abstract: We will discuss recent progress on associating operators to the generalized filter functions of Baggett, Courter, and Merrill related to a generalized multiresolution analysis. This work attempts to generalize work of O. Bratteli and P. Jorgensen for the classical case of low and high pass filters associated to dilation by N. This work is joint with Lawrence Baggett, Palle Jorgensen, and Kathy Merrill.

(18) Quinto, Todd (equinto@math.tufts.edu)

Title: Practical and mathematical problems in Limited Data Tomography.

Abstract # : 984-92-37

Abstract: Tomography has revolutionized diagnostic medicine, scientific testing, and industrial nondestructive evaluation, and some of the most difficult problems involve limited data, in which some data are missing. This talk will describe two practical problems and give the mathematical background. The first problem, in industrial nondestructive evaluation (joint with Percetics, Inc), uses limited-angle exterior CT to reconstruct a rocket mockup. The second, in electron microscopy (joint with Sidec Technologies), uses limited angle local CT to reconstruct RNA.

(19) Spegle, Darrin (speegled@slu.edu)

Title: Wave packet densities

Joint with Wojciech Czaja, wojtek@math.umd.edu and Gitta Kutyniok, gittak@uni-paderborn.de

Abstract # : 984-42-191

Abstract: Functions which form orthonormal bases (frames, etc.) for $L^2(\mathbb{R}^d)$ when a discrete set W of dilations, translations and modulations are applied to the function are considered. A notion of density related to the Beurling density of this discrete set is developed, and preliminary results on the possible densities of W are obtained. A closely related notion of dimension of W is also considered, where preliminary results are also obtained. Primary results are of the type that describe the discrete sets W for which these wave packets can be constructed.

(20) Sun, Qiyu (qiyu.sun@mail.uh.edu)

Title: Localization of Stability, Frame and Sampling in Fourier Domain.

Abstract # : 984-42-94

Abstract: In this paper, we introduce and study the localization of stability, frame and average sampling properties of the system generated by the integer shifts of finitely many functions in Fourier domain.

(21) Weber, Eric (esw@uwyo.edu)

Title: Orthogonal Frames of Translates.

Abstract # : 984-42-100

Abstract: We consider two frames which arise from the action of a collection of translates. Frames of this form appear in wavelet frames, Gabor frames, and sampling theory. Our goal is to characterize when the frames are orthogonal, i.e. when their analysis operators have orthogonal ranges. We will focus on the case when the translates are not a group, and may not even have any regularity, such as irregular wavelet and Gabor frames.

(22) Weiss, Guido (guido@math.wustl.edu)

Title: Applications to a unified theory of reproducing systems.

Abstract # : 984-42-118

Abstract: In a recent paper the author, together with D. Labate, E. Hernandez and E. wilson, has obtained a unified approach to the study of general reproducing systems. These include wavelets, Gabor systems, wave packets, as well as many other. We shall present how the concept of oversampling applies in this general setting.

(23) Wilson, Edward (enwilson@math.wustl.edu)

Title: Connectivity for tight frame Gabor systems with rational lattices.

Joint with: Demetrio Labate, dlabate@math.wustl.edu

Abstract # : 984-42-93

Abstract: Gabor systems arise by applying lattice translation and lattice modulation operators to a fixed generating function in $L^2(\mathbb{R}^n)$. Without loss of generality, one may assume the translation lattice is the standard integer lattice $\mathbb{Z}^n$. The lattice used for modulations is then $B\mathbb{Z}^n$ for some invertible matrix B. When the entries of B are rational numbers, we show that the generators for any two normalized tight frame (NTF) Gabor systems can be joined by a continuous path in $L^2(\mathbb{R}^n)$ with each point on the path generating a NTF Gabor system. Our method extends to matrices which are partly rational in the sense that $\mathbb{Z}^n + B\mathbb{Z}^n$ is not dense in $\mathbb{R}^n$. The method breaks down in the completely irrational case when the sum is dense.