

Midterm Two, Thursday, March 6, 2008
Brendan Owens, 1552

Name: _____

[1] 25	[2] 25	[3] 25	[4] 25	TOTAL

Please leave these boxes blank!

There are 4 problems on this exam. Each problem is worth 25 points, for a total of 100 points. You must show **all** your work to receive credit. **Important: No books, calculators, or notes are allowed.** Please read each question carefully, show all work, and check afterwards that you have answered all of each question correctly. Write **one** clear answer with a coherent derivation. Good luck!

Double angle identities:

$$\cos^2 \theta = \frac{1 + \cos(2\theta)}{2}$$
$$\sin^2 \theta = \frac{1 - \cos(2\theta)}{2}$$

Names of some convergence/divergence tests:

- divergence test
- integral test
- direct comparison test
- limit comparison test
- alternating series test
- ratio test

Taylor series of $f(x)$:
$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n$$

[1]

(a) Evaluate the integral $\int_0^{\infty} x^{-5/2} dx$ or say if it diverges.

(b) Determine whether or not the integral $\int_0^{\infty} \frac{1}{\sqrt{x^5 + 2}} dx$ converges.

[1] (25 pts)

Please leave blank!

[2] For each of the following series, say whether it is convergent or divergent. (Justify your answer and name any tests that you use.)

(a)

$$\sum_{n=1}^{\infty} \frac{(-2)^{n+1}}{3^{n+2}}$$

(b)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

(c)

$$\sum_{n=1}^{\infty} \frac{n^2}{(2n+1)!}$$

[2] (25 pts)

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Please leave blank!

[3] Show that

$$S = \frac{1}{2} - \frac{1}{2} + \frac{1}{3} - \frac{1}{3} + \frac{1}{4} - \frac{1}{4} + \dots$$

converges by computing partial sums. Does it converge absolutely?

[3] (25 pts)

Please leave blank!

[4]

- (a) For which values of x does the power series $\sum_{n=1}^{\infty} \frac{2^n x^n}{n}$ converge?
- (b) Find the Taylor series for e^x centered at 1.

[4] (25 pts)

Please leave blank!