Review Sheet for the Final Exam
Brendan Owens, Math 1552, Section 1

- The Final Exam is on Monday May 5th from 5:30pm to 7:30pm in our usual classroom (Lockett 116).
- It will probably contain 10 questions, of which at least two questions will be on the material for each of Exam 1, Exam 2, and Exam 3 and at least three questions will be on the material since Exam 3.
- At least one question will be taken from one of the three exams, and at least one question will be taken from the sample problems on this sheet. Some parts of problems will be taken from the suggested textbook problems on the class webpage.
- The front page or pages of the Final will include all information from the front pages of Exams 1, 2 and 3 (trapezoid error bound, double angle identities, etc.) It will also include the following formula for curvature:

$$\kappa(t) = \frac{\|\vec{r}'(t) \times \vec{r}''(t)\|}{\|\vec{r}'(t)\|^3}.$$

Here is a brief summary of the material covered in the course:

Material for Exam 1

From Chapter 7: Trapezoid rule and trapezoid rule error bound; integration by parts; trig. integrals; trig substitution; partial fractions.

Material for Exam 2

From Chapter 7: Improper integrals.

From Chapter 10: Sequences, limits of sequences; series and partial sums; geometric series; telescoping series; divergence test; integral test; p-series; comparison tests (direct and limit); alternating series test and alternating series estimation; ratio test; absolute and conditional convergence; power series – radius and interval of convergence, differentiation and integration, examples; Maclaurin and Taylor series.

Material for Exam 3

From Chapter 10: Binomial series, application of Taylor series and polynomials.

From Chapter 11: parametric curves in $\mathbb{R}^2$ – examples, arclength and speed, derivatives and tangent lines; polar coordinates and polar graphs; tangent lines, arclength, and area of polar graphs.

From Chapter 12: 3-dimensional coordinates ($\mathbb{R}^3$); vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$ – addition, scalar mult., length, unit vectors, dot product and angles between vectors, projections, cross product, area of parallelogram, volume of parallelopiped.

Material since Exam 3

From Chapter 12: Lines and planes in $\mathbb{R}^3$; quadric surfaces and traces.

From Chapter 13: parametric curves in $\mathbb{R}^3$ and vector-valued functions, velocity and acceleration vectors; calculus of vector-valued functions; tangent line, arclength, curvature.

From Chapter 14: partial derivatives.

Sample problems

NOTE: These questions are meant to be helpful practice problems. They are NOT a comprehensive review. You should also review all previous homeworks and midterms carefully. Use the suggested problems from the textbook as well as the examples given in the textbook as extra practice.

1. Evaluate the integrals:

(i) $\int xe^{-x}\,dx$ (ii) $\int e^x \sin x\,dx$ (iii) $\int_1^3 \frac{\ln x}{x^2}\,dx$

2. Evaluate the integrals:

(i) $\int_\pi^{2\pi} \sin^2 x\,dx$ (ii) $\int \sin^7 x \cos^3 x\,dx$ (iii) $\int \tan^5 x \sec^3 x\,dx$

3. Evaluate the integrals:

(i) $\int \frac{x^3}{\sqrt{25-x^2}}\,dx$ (ii) $\int \frac{dx}{(x^2+4)^2}\,dx$ (iii) $\int \frac{dx}{x^2\sqrt{x^2-4}}$

4. Evaluate the integrals using the method of partial fractions:

(i) $\int \frac{1}{(t+4)(t-1)}\,dx$ (ii) $\int \frac{x^3+3x}{x^2-x-6}\,dx$

5. How many subintervals are required to guarantee that the Trapezoid Rule approximation for the integral

$$\int_0^1 e^{-x^2}\,dx$$

is accurate to within $\frac{1}{1000}$?

6. Determine whether each integral converges or diverges. If it converges, evaluate the integral.

(i) $\int_{-\infty}^0 \frac{1}{3x+2}\,dx$ (ii) $\int_{-\infty}^\infty xe^{-x^2}\,dx$ (iii) $\int_0^\infty \sin^2 x\,dx$

7. Say whether each of the following series is convergent. If so, find the sum.

(i) $\sum_{n=0}^\infty \left(\frac{2^{n+1}}{3^n} - \frac{3^n}{4^n}\right)$ (ii) $\sum_{n=1}^\infty \ln\left(\frac{n+1}{n}\right)$

8. Test the following series for convergence.

(i) $\sum_{n=1}^\infty e^n$ (ii) $\sum_{n=1}^\infty e^{-n}$ (iii) $\sum_{n=1}^\infty \frac{1}{3n+1}$.

9. Determine whether the following series are convergent:

(i) $\sum_{n=1}^\infty \frac{n}{n^3+1}$ (ii) $\sum_{n=1}^\infty \frac{1}{2^n-1}$ (iii) $\sum_{n=1}^\infty \frac{n^3}{3^n}$.

10. Show that the following series converges, and determine how many terms are sufficient to estimate the sum to within 0.0001.

$$\sum_{n=1}^\infty \frac{(-1)^n}{n^4}$$

11. Say whether the series in the previous problem is divergent, conditionally convergent, or absolutely convergent.

12. Consider the series $\sum_{n=1}^{\infty} a_n$, where $a_n = \dfrac{(-1)^n}{n}$.

(i) Find $\lim_{n\to\infty} a_n$.

(ii) If s_n represents the partial sum obtained by adding the first n terms of the series, does $\lim_{n\to\infty} s_n$ exist?

(iii) Is the series convergent?

(iv) Is the series absolutely convergent?

13. For which values of x do the following series converge?

(i) $\sum_{n=1}^{\infty} (-1)^n \dfrac{x^n}{n^2 7^n}$ (ii) $\sum_{n=1}^{\infty} \dfrac{(x+2)^n}{n4^n}$ (iii) $\sum_{n=0}^{\infty} \dfrac{2^n(x-2)^n}{(n+2)!}$

14. (i) What function is represented by the power series

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots ,$$

and for which values of x does it converge? Hint: this is a geometric series.

Using part (i), find the Maclaurin series and its radius of convergence for

(ii) $\dfrac{1}{1+x}$ and then $\ln(1+x)$ and $\dfrac{1}{(1+x)^2}$

(iii) $\dfrac{1}{1+x^2}$ and then $\tan^{-1} x$.

Hint: use term-by-term differentiation and integration.

15. (i) Find the Maclaurin series for $f(x) = e^x$.

(ii) Using the series from part (i), find a sum of fractions which approximates e^{-1} to within 0.01. (Hint: use alternating series estimation.) You do not need to simplify the sum.

16. (i) Find the Maclaurin series for $\sin x$ and $\cos x$.

(ii) Write down the 4th degree Maclaurin polynomials for each of $\sin x$ and $\cos x$.

(iii) Use 4th degree Maclaurin polynomials to estimate $\sin 1$ and $\cos 1$.

17. (i) Find the Maclaurin series for $f(x) = x^2 \cos(2x)$. Hint: first find the series for $\cos x$, use it to find the series for $\cos(2x)$, then multiply by x^2.

(ii) For the same function as in part (i), the Taylor series centered at π is a power series $\sum_{n=0}^{\infty} c_n(x-\pi)^n$. Find c_0 and c_1.

18. Write parametric equations for a curve which traverses the circle $x^2 + y^2 = 9$ once clockwise as t goes from 0 to 2π.

19. Find an equation of the tangent to the curve $x = 2\sin t, y = \sin 2t$ when $t = \frac{\pi}{6}$.

20. Find dy/dx for the curve $x = t - e^t, y = t + e^{-t}$. Where does the curve have a horizontal tangent? Where does the curve have a vertical tangent?

21. Sketch a graph of each of the following polar curves:

    (i) $r = 5$ (ii) $\theta = \frac{\pi}{4}$ (iii) $r = 3\cos 5\theta$ (iv) $r = \sin 2\theta$ (v) $r = 1 + \cos\theta$

22. For the polar curve given in part (iii) of the previous problem, write down an integral representing the area of one leaf of the rose and an integral representing the arclength of the same leaf (but do not evaluate either one).

23. If $\vec{v} = \langle 1, 2, -4 \rangle$ and $\vec{w} = \langle 1, 0, 5 \rangle$, compute the following.

    (i) $5\vec{v} + 2\vec{w}$ (ii) $\vec{v} \cdot \vec{w}$ (iii) $\vec{v} \times \vec{w}$ (iv) $\|\vec{v}\|$

    (iv) The area of the parallelogram spanned by $\vec{v}$ and $\vec{w}$.

    (v) The unit vector pointing in the same direction as the vector $\vec{v}$.

    (vi) The angle between $\vec{v}$ and $\vec{w}$.

    (vii) $\|\text{proj}_{\vec{v}}\vec{w}\|$ (viii) $\text{proj}_{\vec{v}}\vec{w}$ (note this is also written $\vec{w}_{\|}$)

24. If $\vec{u} = \langle 1, 0, 5 \rangle$, find the volume of the parallelepiped spanned by $\vec{u}$ and the vectors $\vec{v}$ and $\vec{w}$ from the previous problem.

25. Find the parametric equations of the line through the point $P = (2, 5, -6)$ parallel to the vector $\vec{v} = \langle 1, 2, 3 \rangle$.

26. Find the area of the triangle with vertices $P = (2, 0, 4), Q = (-3, 1, 1), R = (-1, 0, 6)$.

27. Find the equation of the plane through the same points P, Q, R as in the previous problem.

28. Find the equation of the plane parallel to the plane $x - y - z = 0$ and containing the line $x = 5 - 3t, y = 2 + t, z = -2t$.

29. Find the distance from the point $P = (2, 5, 0)$ to the plane $x + z = 7$.

30. Find some x-, y-, and z-traces for each of the given surfaces. Identify each trace as a point, line, parabola, circle, ellipse, or hyperbola. Identify the surface as one of the six types listed on p. 724–725.

    (i) $4x^2 + 9y^2 + z^2 = 25$ (ii) $4x^2 + 9y^2 - z^2 = 0$ (iii) $z = x^2 + y^2$ (iv) $z = x^2 - y^2$

31. For the curve in $\mathbb{R}^3$ given by the vector-valued function $\vec{r}(t) = \langle t, t^2, t^3 \rangle$

    (i) give parametric equations for the tangent line to the curve at $t = 1$

    (ii) write down an integral representing the arclength of the curve from $t = 0$ to $t = 1$ (do not evaluate the integral).

32. For the curve given in the previous problem, compute the curvature at $t = 0$. Compute the function $\kappa(t)$.

33. If $f(x) = \sin(xy^2)$ find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$. Find $\dfrac{\partial^2 f}{\partial x \partial y}$ and $\dfrac{\partial^2 f}{\partial y \partial x}$ and verify that thes two "mixed partials" are equal.