Print Your Name Here: _____

- **Show all work** in the space provided. We can give credit *only* for what you write! *Indicate clearly if you continue on the back side*, and write your name at the top of the scratch sheet if you will turn it in for grading.
- **No** books or notes (paper or electronic) or communication devices (smart/cell phones, internet-connected devices such as laptops, tablets, or I-watches) are allowed. A scientific calculator (*not capable* of graphing or symbolic calculations) is allowed—but it is not needed. If you use a calculator, you *must still write out all operations performed* on the calculator. Do not replace precise answers such as $\sqrt{2}$, $\frac{1}{3}$, or π with decimal approximations. Keep your eyes on your own paper!
- There are **five (5)** problems and the *Maximum total score* = 100.

1. (15) Evaluate the limit or say why it does not exist: $\lim_{t\to 2} \dfrac{t^2+t-6}{t^2-4}$.

2. (15) Give a formal proof that $\lim_{x\to 2} 3x+4 = 10$ as follows. Let $\epsilon > 0$. *Find* a corresponding $\delta > 0$ such that $0 < |x-2| < \delta$ implies $|(3x+4)-10| < \epsilon$.

3. (20) Suppose $f(x) = \begin{cases} c - x^2, & x < 2 \\ d, & x = 2 \\ 2x - c, & x > 2. \end{cases}$ Find the values of c and d that make f continuous at $x = 2$.

4. (20) Find $\lim_{x \to \infty} \left(\sqrt{x^2 + 4x} - \sqrt{x^2 + x} \right)$ or explain why it does not exist.

5. (30) Let $f(x) = \dfrac{1}{x},\ x \neq 0$.

a. (20) Find $f'(x)$ by evaluating $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ if it exists.

b. (10) Write an equation for the tangent line to the graph of $y = f(x)$ at the point $\left(2, \dfrac{1}{2}\right)$.

Solutions

1. Factoring the numerator and denominator we see that $\lim_{x\to 2} \dfrac{t^2+t-6}{t^2-4} = \lim_{x\to 2} \dfrac{t+3}{t+2} = \dfrac{5}{4}$.

2. Solving the inequality $|(3x+4)-10| < \epsilon$ for $|x-2|$ we find that it is necessary and sufficient that $|x-2| < \dfrac{\epsilon}{3}$. Thus we can use any $\delta > 0$ such that $\delta \leq \dfrac{\epsilon}{3}$.

3. For continuity at $x = 2$ we need both $\lim_{x\to 2-} f(x) = d$ and $\lim_{x\to 2+} f(x) = d$. Thus $c-4 = d$ and $4-c = d$. Hence $d = 0$ and $c = 4$.

4. Rationalizing the numerator and then dividing numerator and denominator by x shows that the limit is $\dfrac{3}{2}$. There were many errors with signs in expanding the difference of two squares because of laxity in using parentheses. Also, be careful to use the equal sign "=" only if you really mean that the left side equals the right side. It is incorrect to write "=" as if it were a general connective between one step and the next step.

5.

a. $f'(x) = \lim_{h\to 0} \dfrac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h\to 0} \dfrac{-1}{x(x+h)} = \dfrac{-1}{x^2}$.

b. $x + 4y = 4$ or any equivalent equation. Be aware that the equation of a straight line is a *linear* equation!

Class Statistics

% Grade	Test#1	Test#2	Test#3	Test 4	Test 5	Final Exam	Final Grade
90-100 (A)	12						
80-89 (B)	10						
70-79 (C)	4						
60-69 (D)	5						
0-59 (F)	1						
Test Avg	83.3%	%	%	%	%	%	%