Print Your Name Here: _____

- **Show all work** in the space provided. We can give credit *only* for what you write! *Indicate clearly if you continue on the back side*, and write your name at the top of the scratch sheet if you will turn it in for grading.
- **No** books or notes (paper or electronic) or communication devices (smart/cell phones, internet-connected devices such as laptops, tablets, or I-watches) are allowed. A scientific calculator (*not capable* of graphing or symbolic calculations) is allowed—but it is not needed. If you use a calculator, you *must still write out all operations performed* on the calculator. Do not replace precise answers such as $\sqrt{2}$, $\frac{1}{3}$, or π with decimal approximations. Keep your eyes on your own paper!
- There are **five (5)** problems and the *Maximum total score* $= 100$.

1. (20) Differentiate each of the following functions:

a. $g(x) = (x^2 + 1)e^{2x}$.

b. $y = \dfrac{x}{x^2 + 1}$.

2. (20) Differentiate the following functions:

a. $f(x) = \dfrac{\sin x}{1 + \cos x}$.

b. $g(x) = \tan(e^{\sin x})$.

3. (20)

a. Use *implicit differentiation* to find $y' = \frac{dy}{dx}$ if $\ln(xy) = x^2 + y^2$.

b. Use logarithmic differentiation to find y' in terms of x and y if $y = x^{2x}$.

4. (20) A 10 foot ladder leans against a vertical wall with the top of the ladder y feet up and the foot of the ladder x feet from the wall on horizontal ground. If the foot of the ladder slides away from the wall at 2 feet per second, find the velocity $\frac{dy}{dt}$ of the top point on the ladder in terms of x.

5. (20) Let $f(x) = xe^{-x}$ on the interval $[0, 2]$. Find the absolute maximum value and the absolute minimum value of f on the given interval. (Show your work and no approximations!)

Solutions

1.

a. By the product formula $g'(x) = 2xe^{2x} + (x^2+1)2e^{2x} = 2(x^2+x+1)e^{2x}$. It is helpful to present answers in factored form when it is possible to do so.

b. Taking care to notice the asymmetry of the quotient formula, $y' = \dfrac{1-x^2}{(1+x^2)^2}$. Be careful with the algebra!

2.

a. $f'(x) = \dfrac{1}{1+\cos x}$.

b. $g'(x) = (\sec^2 e^{\sin x})e^{\sin x}\cos x$.

3.

a. $y' = \dfrac{2x^2y - y}{x - 2xy^2}$.

b. $y' = 2y(1+\ln x) = 2x^{2x}(1+\ln x)$, written either way.

4. $\dfrac{dy}{dt} = \dfrac{-2x}{\sqrt{100-x^2}}$. Note that the top of the ladder has velocity approaching $-\infty$ as $x \to 10$, violating the special theory of relativity!

5. The maximum value is $f(1) = \frac{1}{e}$ and the minimum value is $f(0) = 0$. Note that $\dfrac{1}{e} > \dfrac{2}{e^2}$ since $\dfrac{2}{e} < 1$. No calculator is needed to see this.

Class Statistics

% Grade	Test#1	Test#2	Test#3	Test 4	Test 5	Final Exam	Final Grade
90-100 (A)	12	16					
80-89 (B)	10	8					
70-79 (C)	4	3					
60-69 (D)	5	2					
0-59 (F)	1	3					
Test Avg	83.3%	84.5%	%	%	%	%	%