Print Your Name Here: _____

- **Show all work** in the space provided. We can give credit *only* for what you write! *Indicate clearly if you continue on the back side*, and write your name at the top of the scratch sheet if you will turn it in for grading.
- **No** books or notes (paper or electronic) or communication devices (smart/cell phones, internet-connected devices such as laptops, tablets, or I-watches) are allowed. A scientific calculator (*not capable* of graphing or symbolic calculations) is allowed—but it is not needed. If you use a calculator, you *must still write out all operations performed* on the calculator. Do not replace precise answers such as $\sqrt{2}$, $\frac{1}{3}$, or π with decimal approximations. Keep your eyes on your own paper!
- There are **five (5)** problems and the *Maximum total score* = 100.

1. (20) Use l'Hospital's Rule to evaluate the following limits.

a. $\lim_{x\to\infty} \dfrac{\ln x}{\sqrt{x}}$

b. $\lim_{x\to 0} \dfrac{\sin x - x}{x^3}$

2. (20) Find the *most general* antiderivative $F(x)$ of the given function and check by differentiating.

a. $f(x) = x^2 - x^{\frac{1}{2}}$

b. $f(x) = e^{2x} + \sec^2 x$

3. (30) A farmer wishes to enclose with fencing a rectangular field with a *total area* of 1000 square meters, and then *partition* the field into two sections with a fence parallel to one side of the rectangle. Find the *minimum length* s of fencing that is needed.

4. (30) Consider the function $f(x) = \dfrac{x+1}{x^2}$, $x \neq 0$. Use the formula for derivatives of quotients (making use of cancellations for simplification) to find the following information.

a. horizontal asymptote(s): _____

b. vertical asymptote(s):_____

c. f increasing on:_____

d. f decreasing on:_____

e. local maximum at:_____

f. local minimum at:_____

g. concave up on:_____

h. concave down on:_____

i. point(s) of inflection at:_____

j. sketch!

Solutions

1. Please do not write that a limiit equals $\frac{0}{0}$ or $\frac{\infty}{\infty}$ since these are undefined operations. Do not omit the limit from an equality where it is needed because the resulting equation would be a false statement. Please see the examples of correct solutions below.

a. $\lim_{x\to\infty} \dfrac{\ln x}{\sqrt{x}} = \lim_{x\to\infty} \dfrac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim_{x\to\infty} \dfrac{2}{\sqrt{x}} = 0.$

b. $\lim_{x\to 0} \dfrac{\sin x - x}{x^3} = \lim_{x\to 0} \dfrac{\cos x - 1}{3x^2} = \lim_{x\to 0} \dfrac{-\sin x}{6x} = \lim_{x\to 0} \dfrac{-\cos x}{6} = \dfrac{-1}{6}.$

2. Don't forget the arbitrary additive constants!

a. $F(x) = \dfrac{x^3}{3} - \dfrac{2}{3}x^{\frac{3}{2}} + C$

b. $F(x) = \dfrac{1}{2}e^{2x} + \tan x + C$

3. If x and y are dimensions of the rectangle we can write $s = 3x + 2y$ and $xy = 1000$, so that $s = 3x + \dfrac{2000}{x}$. Since $s \to \infty$ as $x \to 0+$ and as $x \to \infty$, the minimum occurs where $s'(x) = 0$. Thus $x = 10\sqrt{\dfrac{20}{3}} = \dfrac{20}{3}\sqrt{15}$ and $y = 100\sqrt{\dfrac{3}{20}} = 10\sqrt{15}$. The minimum total length of fencing needed is $40\sqrt{15}$ meters or the equivalent.

4. For graphing problems it is necessary to find f' and f'' *very* carefully!

a. horizontal asymptote(s): $y = 0$

b. vertical asymptote(s): $x = 0$

c. f increasing on: $(-2, 0)$

d. f decreasing on: $(-\infty, -2),\ (0, \infty)$

e. local maximum at: none

f. local minimum at: $x = -2,\ y = -\frac{1}{4}$

g. concave up on: $(-3, 0),\ (0, \infty)$

h. concave down on: $(-\infty, -3)$

i. point(s) of inflection at: $x = -3,\ y = -\frac{2}{9}$

j. See Figure 1 below.

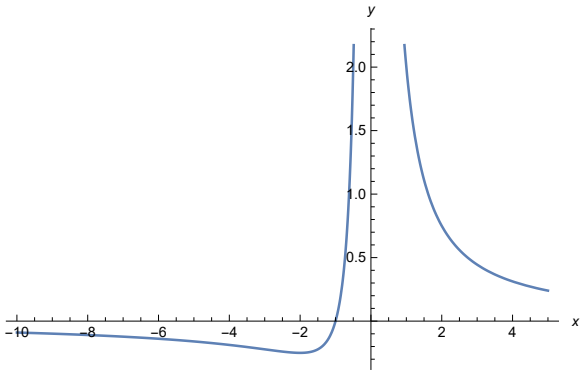

Figure 1: $y = \frac{x+1}{x^2}$.

Class Statistics

% Grade	Test#1	Test#2	Test#3	Test 4	Test 5	Final Exam	Final Grade
90-100 (A)	12	16	13				
80-89 (B)	10	8	7				
70-79 (C)	4	3	6				
60-69 (D)	5	2	2				
0-59 (F)	1	3	2				
Test Avg	83.3%	84.5%	84.7%	%	%	%	%