Print Your Name Here: ______________________________

- **Show all work** in the space provided. We can give credit *only* for what you write! *Indicate clearly if you continue on the back side*, and write your name at the top of the scratch sheet if you will turn it in for grading.
- **No** books or notes (paper or electronic) or communication devices (smart/cell phones, internet-connected devices such as laptops, tablets, or I-watches) are allowed. A scientific calculator (*not capable* of graphing or symbolic calculations) is allowed—but it is not needed. If you use a calculator, you *must still write out all operations performed* on the calculator. Do not replace precise answers such as $\sqrt{2}$, $\frac{1}{3}$, or π with decimal approximations. Keep your eyes on your own paper!
- There are **four (4)** problems and the *Maximum total score* = 100.

1. Evaluate and simplify using the appropriate part of the Fundamental Theorem of Calculus:

a. (15) $\dfrac{d}{dx}\displaystyle\int_1^{e^{3x}} \ln t\,dt$

b. (10) $\displaystyle\int_0^2 x^2 + 2^x\,dx$

2. Evaluate the following indefinite and definite integrals, as indicated.

a. (10) $\displaystyle\int \frac{x + \sqrt[3]{x} + 1}{x}\,dx$

b. (15) $\displaystyle\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-x^2}}\,dx$

3. Evaluate by the method of substitution:

a. (10) $\int (x^2 + 2x)^5 (x + 1)\, dx$

b. (15) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x}\, dx$

4. (25) Sketch the region enclosed between the two curves $x = y^2$ and $x = 2 - y^2$. Find the area of the region, making a judicious choice to integrate with respect to x or to y.)

Solutions

1. Don't forget to use the Chain Rule in part (a)! Use $u = e^{3x}$.

a. $\frac{d}{dx}\int_1^{e^{3x}} \ln t\,dt = \frac{d}{du}\int_1^{u} \ln t\,dt \left(\frac{du}{dx}\right) = (\ln u)\left(\frac{du}{dx}\right) = 9xe^{3x}.$

b. $\int_0^2 x^2 + 2^x\,dx = \frac{8}{3} + \frac{3}{\ln 2}.$

2. Beware: $\int \frac{f}{g} \neq \frac{\int f}{\int g}$!!!!

a. $\int \frac{x + \sqrt[3]{x} + 1}{x}\,dx = x + 3\sqrt[3]{x} + \ln|x| + C.$

b. $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-x^2}}\,dx = \frac{\pi}{6}$. If you don't remember the derivatives of the inverse trigonometric functions, either derive them as shown in class or use the Pythagorean theorem to find a substitution as shown in class.

3.

a. $\int (x^2+2x)^5(x+1)\,dx = \frac{(x^2+2x)^6}{12} + C$. Substitute $u = (x^2 + 2x)$.

b. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x}\,dx = \frac{1}{2}\ln 2$. Substitute $u = \sin x$.

4. $A = \int_{-1}^{1} 2 - 2y^2\,dy = \frac{8}{3}.$

Class Statistics

% Grade	Test#1	Test#2	Test#3	Test 4	Test 5	Final Exam	Final Grade
90-100 (A)	12	16	13	13			
80-89 (B)	10	8	7	6			
70-79 (C)	4	3	6	4			
60-69 (D)	5	2	2	3			
0-59 (F)	1	3	2	3			
Test Avg	83.3%	84.5%	84.7%	82.6%	%	%	%