Print Your Name Here: _____

- **Show all work** in the space provided. We can give credit *only* for what you write! *Indicate clearly if you continue on the back side*, and write your name at the top of the scratch sheet if you will turn it in for grading.
- **No** books or notes (paper or electronic) or communication devices (smart/cell phones, internet-connected devices such as laptops, tablets, or I-watches) are allowed. A scientific calculator (*not capable* of graphing or symbolic calculations) is allowed—but it is not needed. If you use a calculator, you *must still write out all operations performed* on the calculator. Do not replace precise answers such as $\sqrt{2}$, $\frac{1}{3}$, or π with decimal approximations. Keep your eyes on your own paper!
- There are **five (5)** problems and the *Maximum total score* = 100.

1. (20) Let R be the region bounded by the graphs of $y = x + 2$ and $y = x^2$, $-1 \leq x \leq 2$. *Set up* the definite integrals (including upper and lower limits of integration) for the *volumes* of each of the following two solids of revolution. *Do **not** evaluate the integrals **or** simplify the integrand.*

a. (10) Revolve the region R about the x-axis. (Use the method of washers.)

b. (10) Revolve the region R about the line $x = -1$. (Use the method of shells.)

2. (20) A 6 lb force will stretch a spring 3 feet beyond its natural (equilibrium) length. Find the spring constant k and the amount of work that was done in stretching the spring to that length?

3. (20) Find the *average* value of the function $f(x) = \dfrac{\ln x}{x}$ over the interval $1 \leq x \leq 10$.

4. (20) Find the *arc length* of the graph of $y = 1 + 2x^{\frac{3}{2}},\ 0 \leq x \leq 1$.

5. (20) Find the *surface area* generated by revolving the graph of $y = x^3,\ 0 \leq x \leq 1$ around the x-axis.

Solutions

1. There was much confusion over the method of shells and the method of washers.

a. $V = \pi \displaystyle\int_{-1}^{2} (x+2)^2 - x^4 \, dx$

b. $V = 2\pi \displaystyle\int_{-1}^{2} (x+1)(x+2-x^2) \, dx.$

2. It is important that the force $F = kx$ is not constant! The spring constant $k = 2$ lbs/ft since $3k = 6$. Thus $W = \displaystyle\int_0^3 2x \, dx = 9$ ft-lbs of work.

3. The average is $\dfrac{1}{9}\displaystyle\int_1^{10} \frac{\ln x}{x} \, dx = \frac{(\ln 10)^2}{18}$. Note that $(\ln 10)^2 \neq \ln 10^2$. Parentheses are important!

4. $L = \displaystyle\int_0^1 \sqrt{1+9x} \, dx = \frac{2}{27}(10\sqrt{10} - 1)$. Many errors resulted from failure to write out the change of variables (substitution) needed for integration.

5. $S = 2\pi \displaystyle\int_0^1 x^3\sqrt{1+9x^4} \, dx = \frac{\pi}{27}(10\sqrt{10} - 1)$. Note that the radius of revolution is x^3. Never try to fake the antidifferentiation.

Class Statistics

% Grade	Test#1	Test#2	Test#3	Test 4	Test 5	Final Exam	Final Grade
90-100 (A)	12	16	13	13	12		
80-89 (B)	10	8	7	6	6		
70-79 (C)	4	3	6	4	6		
60-69 (D)	5	2	2	3	3		
0-59 (F)	1	3	2	3	2		
Test Avg	83.3%	84.5%	84.7%	82.4%	82.4%	%	%