I have read, understood, and complied with the instructions in the box below. Legible

***Signature* and LSU ID #:** _____

- Download a copy of this test. If you have a device with a stylus that can write directly on the pdf file, please use it. Just click on "comment" in the right-hand margin and then click on the icon for a stylus that appears at the top, and you should be able write, and erase using the icon for an eraser at the top. Otherwise, print a copy of this test on 8.5 by 11 inch letter size paper. If no printer is available, make a hand-written facsimile. Be sure to copy and sign the statement above even if you make a hand-written facsimile. But you do not need to hand-copy this large box of instructions. Do copy each question statement and number however on your facsimile.
- **Show *All* Work** in the space provided. Grading is based on the correctness of the work shown to justify the answers. We can give credit *only* for what you write! *Indicate clearly if you continue a problem on a second page.* There are 4 problems.
- ***You may use your text book, Zoom recordings of our class meetings, your class notes, and your homework!*** *However, no other sources or communication devices may be used.* ***All work must be your own.*** If you use a calculator, you *must still write out all operations performed* on the calculator. *Do not replace* precise answers, such as $\sqrt{2}$, π, or $\cos\frac{\pi}{7}$ with decimal approximations. *Make all obvious simplifications.* Submit only your own work!
- This is a take-home test on an *honor system.* You may take as much time as you like, but **I must receive your completed test by email no later than 12:30 PM on Saturday, March 6.** If you have no device that scans your work directly to a single pdf file, then photograph your pages *in the correct order* with your phone, being sure to *orient all pages the same way,* and save as jpeg, then try this please: put the jpeg files into your computer, highlight the whole group of pictures, right click PRINT and then select PRINT TO PDF. That way I can receive a multipage PDF file which is possible to grade in a way you will be able to read later. Email that file to me **rich@math.lsu.edu** as soon as you are ready but no later than Saturday, March 6, at 12:30 PM. *These instructions express my trust and confidence in your integrity and good character.*

Before you send me your pdf file containing all your pages as one single file, with the problems in the correct order, and please make sure everything is legible. Use a sufficiently dark writing instrument for your test and make sharp, clear images, so I can read them. I simply cannot grade what I cannot read. Thank you for your consideration in this!

1. (40) Let $\mathcal{C}$ be the curve defined by the *parametric equations* $x = \cos t$ and $y = 2\sin t$, $0 \le t \le 2\pi$.

a. (15) Find $\dfrac{dy}{dx}$ **as a function of** t**.**

b. (5) Use the result of part (a) to find the *slope* of the straight line tangent to $\mathcal{C}$ at the point corresponding to $t = \dfrac{\pi}{4}$.

c. (15) Use the result of part (a) to Find $\dfrac{d^2y}{dx^2}$ **as a function of** t**.**

d. (5) Use the result of part (c) to find the interval $a < t < b$ on which the curve $\mathcal{C}$ is *concave down*.

2. (20) Find the area of the region R that lies *inside* the circle given by the *polar equation* $r = 2\sin\theta$ but *outside* the circle $r = 1$. (You will need to find the values of θ at which the two circles intersect. A sketch is helpful.)

3. (20) Consider the conic section with polar equation $r = \dfrac{3}{1-2\cos\theta}$, $\theta \neq \pm\frac{\pi}{3}$.

a. (5) Find the eccentricity e and name the type of conic section.

b. (5) Remembering that the origin is at a focus F, find the *distance* d from F to the *directrix line* D, and a *rectangular (i.e. an* $x, y-$*) equation* for D.

c. (5) Find polar coordinates for the vertices (the points where the polar (i.e. $x-$)axis crosses the graph). (Hint: remember that r can be negative and what this means graphically.)

d. (5) Find the points on the graph corresponding to $\theta = \pm\frac{\pi}{2}$ and make a rough sketch of the conic section, labeling the points mentioned in parts (c) and (d).

4. (20)

a. (7) Express the endless decimal $0.\overline{09} = 0.090909\ldots$ as a ratio of two whole numbers. (Hint: Interpret the endless decimal as the sum of a geometric series.)

b. (7) Find the sum of the telescoping series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{2n}-\frac{1}{2n+2}\right)$.

c. (6) Find all values of x for which $\displaystyle\sum_{n=1}^{\infty}2^n x^n$ converges.