I have read, understood, and complied with the instructions in the box below. Legible

Signature and **LSU ID #:** _____

- Download a copy of this test. If you have a device with a stylus that can write directly on the pdf file, please use it. Just click on "comment" in the right-hand margin and then click on the icon for a stylus that appears at the top, and you should be able write, and erase using the icon for an eraser at the top. Otherwise, print a copy of this test on 8.5 by 11 inch letter size paper. If no printer is available, make a hand-written facsimile. Be sure to copy and sign the statement above even if you make a hand-written facsimile. But you do not need to hand-copy this large box of instructions. Do copy each question statement and number however on your facsimile.

- <u>**Show** *All* **Work**</u> in the space provided. Grading is based on the correctness of the work shown to justify the answers. We can give credit *only* for what you write! *Indicate clearly if you continue a problem on a second page.* There are 4 problems.

- **You may use your text book, Zoom recordings of our class meetings, your class notes, and your homework!** *However, no other sources or communication devices may be used.* **All work must be your own.** If you use a calculator, you *must still write out all operations performed* on the calculator. *Do not replace* precise answers, such as $\sqrt{2}$, π, or $\cos\frac{\pi}{7}$ with decimal approximations. *Make all obvious simplifications.* Submit only your own work!

- This is a take-home test on an *honor system*. You may take as much time as you like, but **I must receive your completed test by email no later than 12:30 PM on Saturday, March 6.** If you have no device that scans your work directly to a single pdf file, then photograph your pages *in the correct order* with your phone, being sure to *orient all pages the same way,* and save as jpeg, then try this please: put the jpeg files into your computer, highlight the whole group of pictures, right click PRINT and then select PRINT TO PDF. That way I can receive a multipage PDF file which is possible to grade in a way you will be able to read later. Email that file to me **rich@math.lsu.edu** as soon as you are ready but no later than Saturday, March 6, at 12:30 PM. *These instructions express my trust and confidence in your integrity and good character.*

Before you send me your pdf file containing all your pages as one single file, with the problems in the correct order, and please make sure everything is legible. Use a sufficiently dark writing instrument for your test and make sharp, clear images, so I can read them. I simply cannot grade what I cannot read. Thank you for your consideration in this!

1.   (40) Let $\mathcal{C}$ be the curve defined by the *parametric equations* $x = \cos t$ and $y = 2\sin t$, $0 \le t \le 2\pi$.

  a. (15) Find $\dfrac{dy}{dx}$ **as a function of** t.

  b. (5) Use the result of part (a) to find the *slope* of the straight line tangent to $\mathcal{C}$ at the point corresponding to $t = \dfrac{\pi}{4}$.

  c. (15) Use the result of part (a) to Find $\dfrac{d^2y}{dx^2}$ **as a function of** t.

  d. (5) Use the result of part (c) to find the interval $a < t < b$ on which the curve $\mathcal{C}$ is *concave down*.

2.   (20) Find the area of the region R that lies *inside* the circle given by the *polar equation* $r = 2\sin\theta$ but *outside* the circle $r = 1$. (You will need to find the values of θ at which the two circles intersect. A sketch is helpful.)

3. (20) Consider the conic section with polar equation $r = \dfrac{3}{1 - 2\cos\theta}$, $\theta \neq \pm\frac{\pi}{3}$.

   a. (5) Find the eccentricity e and name the type of conic section.

   b. (5) Remembering that the origin is at a focus F, find the *distance d from F to the directrix line D*, and a *rectangular (i.e. an $x, y-$) equation* for D.

   c. (5) Find polar coordinates for the vertices (the points where the polar (i.e. $x-$)axis crosses the graph). (Hint: remember that r can be negative and what this means graphically.)

   d. (5) Find the points on the graph corresponding to $\theta = \pm\frac{\pi}{2}$ and make a rough sketch of the conic section, labeling the points mentioned in parts (c) and (d).

4. (20)

    a. (7) Express the endless decimal $0.\overline{09} = 0.090909\ldots$ as a ratio of two whole numbers. (Hint: Interpret the endless decimal as the sum of a geometric series.)

    b. (7) Find the sum of the telescoping series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{2n} - \frac{1}{2n+2}\right)$.

    c. (6) Find all values of x for which $\displaystyle\sum_{n=1}^{\infty} 2^n x^n$ converges.

Solutions

1.

    a. $\dfrac{dy}{dx} = -2\cot t, \ t \neq n\pi$

    b. The slope is -2.

    c. $\dfrac{d^2 y}{dx^2} = -2\csc^3 t, \ t \neq n\pi$

    d. The curve $\mathcal{C}$ is concave down on $0 < t < \pi$.

2. $\quad A = \displaystyle\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{1}{2}(4\sin^2\theta) - \frac{1}{2}1^2 \, d\theta = \frac{\pi}{3} + \frac{\sqrt{3}}{2} = \frac{2\pi + 3\sqrt{3}}{6}$

3.

    a. $e = 2$; hyperbola

    b. $d = \frac{3}{2}$; $x = -\frac{3}{2}$ is a rectangular equation for D.

    c. polar coordinates for the vertices: $V_1(1, \pi)$ and $V_2(-3, 0)$, which could also be expressed as $(3, \pi)$.

    d. (5) $(3, \pm\frac{\pi}{2})$. See Figure (1) below. The figure also shows the asymptotes but you don't need to draw these for credit.

4.

    a. $0.\overline{09} = 0.090909\ldots = \displaystyle\sum_{n=1}^{\infty} 9\left(\frac{1}{100}\right)^n = \frac{1}{11}$.

    b. This telescoping series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{2n} - \frac{1}{2n+2}\right) = \frac{1}{2}$.
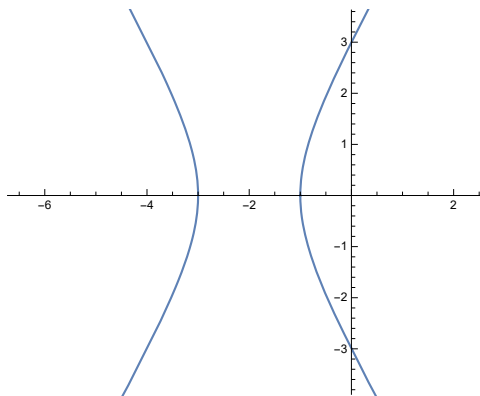
    c. We need $|2x| < 1$, so that $x \in \left(-\frac{1}{2}, \frac{1}{2}\right)$.

Figure 1: Hyperbola for Problem 3.

Class Statistics

Grade	Test#1	Test#2	Test#3	Test#4	Final Exam	Final Grade
90-100 (A)	15	14				
80-89 (B)	4	5				
70-79 (C)	3	3				
60-69 (D)	0	1				
0-59 (F)	0	0				
Test Avg	89.5%	88.91%				