I have read, understood, and complied with the instructions in the box below. Legible

***Signature* and LSU ID #:** ______________________________

- Download a copy of this test. If you have a device with a stylus that can write directly on the pdf file, please use it. Just click on "comment" in the right-hand margin and then click on the icon for a stylus that appears at the top, and you should be able write, and erase using the icon for an eraser at the top. Otherwise, print a copy of this test on 8.5 by 11 inch letter size paper. If no printer is available, make a hand-written facsimile. Be sure to copy and sign the statement above even if you make a hand-written facsimile. But you do not need to hand-copy this large box of instructions. Do copy each question statement and number however on your facsimile.
- **Show *All* Work** in the space provided. Grading is based on the correctness of the work shown to justify the answers. We can give credit *only* for what you write! *Indicate clearly if you continue a problem on a second page.* There are 4 problems.
- ***You may use your text book, Zoom recordings of our class meetings, your class notes, and your homework!*** *However, no other sources or communication devices may be used.* ***All work must be your own.*** If you use a calculator, you *must still write out all operations performed* on the calculator. *Do not replace* precise answers, such as $\sqrt{2}$, π, or $\cos\frac{\pi}{7}$ with decimal approximations. *Make all obvious simplifications.* Submit only your own work!
- This is a take-home test on an *honor system.* You may take as much time as you like, but **I must receive your completed test by email no later than 3 PM on Thursday, April 1.** If you have no device that scans your work directly to a single pdf file, then photograph your pages *in the correct order* with your phone, being sure to *orient all pages the same way,* and save as jpeg, then try this please: put the jpeg files into your computer, highlight the whole group of pictures, right click PRINT and then select PRINT TO PDF. That way I can receive a multipage PDF file which is possible to grade in a way you will be able to read later. Email that file to me **rich@math.lsu.edu** as soon as you are ready but no later than 3 PM on Thursday, April 1. *These instructions express my trust and confidence in your integrity and good character.*

Before you send me your pdf file containing all your pages as one single file, with the problems in the correct order, and please make sure everything is legible. Use a sufficiently dark writing instrument for your test and make sharp, clear images, so I can read them. I simply cannot grade what I cannot read. Thank you for your consideration in this!

Important Note: When you email your completed test back to me, PLEASE put the following in the subject line of your email: 1553_T3_FamilyName_GivenName. This will ensure that your exam is not misplaced into a file of exams from my other class! Thank you.

1. (20) Use the integral test to determine convergence or divergence of the following series.

a. (10) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

b. (10) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

2. (20) Use either a comparison test or a limit comparison test to determine the convergence or divergence of each series.

a. (10) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+7n}{5n^3-4}$

b. (10) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^3-n^2}$

3. (30) Define $f(x) = \sum_{n=1}^{\infty} nx^n$.

a. (10) Use the ratio test to find the radius R of convergence for the power series that defines $f(x)$.

b. (10) Find the interval I of convergence for the power series for $f(x)$. Remember to show how you test the endpoints.

c. (5) Use the power series for $f(x)$ to express $\int_0^{\frac{1}{2}} f(x)\,dx$ as the sum of an infinite series of constants.

d. (5) Use the power series for $f(x)$ to find $\lim_{x\to 0} \dfrac{f(x) - x - 2x^2}{x^3}$.

4. (30) Consider the Taylor series expansion of $f(x) = \ln x$ with base point $a = 3$:

$$\ln x = a_0 + \sum_{n=1}^{\infty} a_n (x-3)^n$$

a. (20) Use the Taylor coefficient formula to find the constant term a_0 and a formula for a_n with $n \geq 1$. Then write out the full Taylor series for $\ln x$ in powers of $x - 3$ using the coefficients that you have found. (You are *not* asked to prove convergence *to* $\ln x$.)

b. (5) Use the result of part (a) and the ratio test to find the radius R of convergence for this Taylor series.

c. (5) Find the interval I of convergence for this Taylor series. Remember to show how you test the left endpoint and the right endpoint for convergence or divergence.

Solutions

1. Caution: ∞ is not a real number. ∞ does not participate in the arithmetic of the real numbers. For example, it is meaningless to write "$\frac{1}{\infty}$" or "$\infty - 1$." If you learn to write your work correctly with limits, your future understanding will be enhanced.

a. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln n}$ diverges because $\dfrac{1}{x\ln x}$ is a decreasing positive function on $[2,\infty)$ and
$$\int_2^{\infty} \frac{1}{x\ln x}\,dx = \lim_{b\to\infty} \int_{\ln 2}^{b} \frac{1}{u}\,du = \lim_{b\to\infty} [\ln b - \ln(\ln 2)] = \infty.$$

b. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ converges because $\dfrac{1}{x(\ln x)^2}$ is a decreasing positive function on $[2,\infty)$ and
$$\int_2^{\infty} \frac{1}{x(\ln x)^2}\,dx = \lim_{b\to\infty} \int_{\ln 2}^{b} \frac{1}{u^2}\,du = \frac{1}{\ln 2} < \infty.$$

2. Caution: Note that n denotes a whole number but the integral test uses a *continuous* variable $x \in \mathbb{R}$, an arbitrary real number.

a. This series diverges by the comparison test since $\dfrac{n^2+7n}{5n^3-4} > \dfrac{n^2}{5n^3-4} > \dfrac{n^2}{5n^3} = \dfrac{1}{5n}$ and $\displaystyle\frac{1}{5}\sum_{n=1}^{\infty}\frac{1}{n}$ is a constant multiple of the divergent harmonic series.

b. This series converges by limit comparison with $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n^3}$ which is a convergent p-series since $p > 1$ and because $\dfrac{\frac{1}{n^3-n^2}}{\frac{1}{n^3}} = \dfrac{n^3}{n^3-n^2} \to 1$ as $n \to \infty$ and $0 < 1 < \infty$.

3.

a. The ratio test yields the limit $|x|$ so $R = 1$.

b. By the nth term test applied to the endpoints, $I = (-1, 1)$.

c. $\displaystyle\int_0^{\frac{1}{2}} f(x)\,dx = \sum_{n=1}^{\infty} \frac{n}{(n+1)2^{n+1}}$

d. $\displaystyle\lim_{x\to 0} \frac{f(x) - x - 2x^2}{x^3} = \lim_{x\to 0} \sum_{n=3}^{\infty} n x^{n-3} = 3.$ Remark: Although it is not required to notice this, $f(x) = \dfrac{x}{(1-x)^2}$.

4.

a. $a_0 = \ln 3$ and $a_n = \dfrac{(-1)^{n+1}}{n3^n}$ with $n \geq 1$. Thus $\displaystyle\ln x = \ln 3 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-3)^n}{n3^n}$.

b. The ratio test yields the limit $\dfrac{|x-3|}{3} < 1$ for convergence. Thus $R = 3$.

c. $I = (0, 6]$ where divergence at $x = 0$ comes from divergence of the harmonic series, and convergence at $x = 6$ comes from the Alternating Series Test.

Class Statistics

Grade	Test#1	Test#2	Test#3	Test#4	Final Exam	Final Grade
90-100 (A)	15	14	9			
80-89 (B)	4	5	6			
70-79 (C)	3	3	3			
60-69 (D)	0	1	3			
0-59 (F)	0	0	3			
Test Avg	89.5%	88.91%	81.67%			