Print Your Name Here: ______________________________

- **Show all work** in the space provided. We can give credit *only* for what you write! *Indicate clearly if you continue on the back side*, and write your name at the top of the scratch sheet if you will turn it in for grading.
- **No** books or notes (paper or electronic) or communication devices (smart/cell phones, internet-connected devices such as laptops, tablets, or I-watches) are allowed. A scientific calculator (*not capable* of graphing or symbolic calculations) is allowed—but it is not needed. If you use a calculator, you *must still write out all operations performed* on the calculator. Do not replace precise answers such as $\sqrt{2}$, $\frac{1}{3}$, or π with decimal approximations. Keep your eyes on your own paper!
- There are **ten (10)** problems and the *Maximum total score* = 200.

1.

a. Find $\lim\limits_{x\to\infty} (\sqrt{9x^2+x} - 3x)$

b. Find $\lim\limits_{x\to 0} \dfrac{\sec x - 1}{x}$

2. Find $f'(x)$ if:

a. $f(x) = \sin^2 3x$

b. $f(x) = 10^x \tan x$

3.

a. Use *implicit differentiation* to find $\frac{dy}{dx}$ if $x^2 + 2y^2 = 6$.

b. Write an equation for the tangent line to the graph of $x^2 + 2y^2 = 6$ at (2,1).

4. A 5 ft tall woman walks away from a 15 ft street lamp at 4 ft/sec. Let x denote her distance from the foot of the lamp post and let s denote the distance of the tip of her shadow from the foot of the lamp post. How fast is the tip of her shadow moving? (Hint: Draw a diagram and use similar triangles.)

5. Let $f(x) = \dfrac{x^2}{x^2+3}$ on $(-\infty, \infty)$. Fill in the following information about the graph.

a. f has horizontal asymptote(s):

b. f has vertical asymptote(s):

c. f is increasing on:

d. f is decreasing on:

e. f is concave up on:

f. f is concave down on:

g. f has a local minimum at the point:

h. f has a local maximum at the point:

i. f has point(s) of inflection at:

j. sketch the graph:

6. Use the appropriate form of the Fundamental Theorem of Calculus:

a. Find $\dfrac{dy}{dx}$ if $y = \displaystyle\int_1^{2x} \frac{t}{1+t^3}\, dt$.

b. Evaluate $\displaystyle\int_1^4 \frac{2+x^2}{\sqrt{x}}\, dx$.

7. Use a substitution to evaluate:

a. $\displaystyle\int e^x \sqrt{1+e^x}\, dx$

b. $\displaystyle\int_1^2 x\sqrt{x-1}\, dx$

8. Find the area of the region enclosed between the two curves $x = 2y^2$ and $x = y^2 + 4$. (Hint: Find the points of intersection of the two curves and sketch the region.)

9. Use the method of your choice to find the volume of the solid obtained by revolving the region bounded by the curves $y = x^2$ and $y = x$ about the x-axis.

10. Find the arclength of the graph of $y = \dfrac{x^2}{4} - \dfrac{1}{2}\ln x$, $1 \leq x \leq 2$. (Hint: Don't panic! The algebra for $\sqrt{1 + (y')^2}$ will simplify very nicely.)

Solutions

1.

a. Rationalize the numerator to show that the limit is $\frac{1}{6}$.

b. This is the derivative of the secant at 0, which is $\sec 0 \tan 0 = 0$.

2.

a. $f'(x) = 6 \sin 3x \cos 3x$.

b. $f'(x) = 10^x(\ln 10 \tan x + \sec^2 x)$. Recall that $\frac{d}{dx}10^x = \frac{d}{dx}e^{x \ln 10} = 10^x \ln 10$. Many errors appear to have resulted from confusing multiplication by $\ln 10$ with division by $\log_{10} e$. The first method includes its own derivation so that there is no risk of misremembering. Rote memorization tends to be a faulty way of learning.

3.

a. $\frac{dy}{dx} = -\frac{x}{2y}$.

b. $x + y = 3$

4. $\frac{s}{15} = \frac{s-x}{5}$, so that $\frac{ds}{dt} = 6$ ft/sec.

5.

a. f has horizontal asymptotes: $y = 1$

b. f has vertical asymptotes: none

c. f is increasing on: $(0, \infty)$

d. f is decreasing on: $(-\infty, 0)$

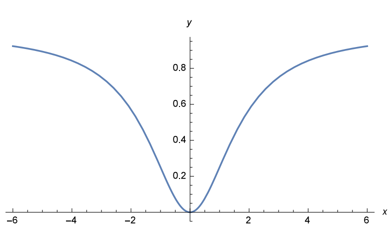
e. f is concave up on: $(-1, 1)$

f. f is concave down on: $(-\infty, -1), (1, \infty)$

g. f has a local minimum at: $(0, 0)$

h. f has a local maximum at: none

i. f has point(s) of inflection at: $(\pm 1, \frac{1}{4})$

j. 

6. Using the two forms of the Fundamental Theorem of Calculus,

a. $\frac{d}{dx}\int_1^{2x} \frac{t}{1+t^3}\,dt = \frac{4x}{1+8x^3}$.

b. $\int_1^4 \frac{2+x^2}{\sqrt{x}}\,dx = \int_1^4 2x^{-\frac{1}{2}} + x^{\frac{3}{2}}\,dx = \frac{82}{5}$.

7.

a. $\int e^x\sqrt{1+e^x}\,dx = \frac{2}{3}(1+e^x)^{\frac{3}{2}} + C$, using $u = 1+e^x$ as a substitution.

b. $\int_1^2 x\sqrt{x-1}\,dx = \int_0^1 (u+1)\sqrt{u}\,du = \frac{16}{15}$, where $u = x-1$ and therefore $x = u+1$.

8. $A = \int_{-2}^{2} y^2+4-2y^2\,dy = \frac{32}{3}$. Integrating with respect to y rather than x replaces two integrals with one.

9. Using washers, $V = \int_0^1 \pi(x^2-x^4)\,dx = \frac{2}{15}\pi$. Or, using shells, $V = \int_0^1 2\pi y(\sqrt{y}-y)\,dy = \frac{2}{15}\pi$.

10. $L = \int_1^2 \frac{x}{2} + \frac{1}{2x}\,dx = \frac{3}{4} + \frac{1}{2}\ln 2$. One needs to be able to recognize $(1+(y')^2)$ as a perfect square.

Class Statistics

% Grade	Test#1	Test#2	Test#3	Test 4	Test 5	Final Exam	Final Grade
90-100 (A)	12	16	13	13	12	8	10
80-89 (B)	10	8	7	6	6	7	10
70-79 (C)	4	3	6	4	6	8	6
60-69 (D)	5	2	2	3	3	3	2
0-59 (F)	1	3	2	3	2	4	2
Test Avg	83.3%	84.5%	84.7%	82.4%	82.4%	79.3%	83.6%