I have read, understood, and complied with the instructions in the box below. Legible

***Signature* and LSU ID #:** ______________________________

- Download a copy of this test. If you have a device with a stylus that can write directly on the pdf file, please use it. Just click on "comment" in the right-hand margin and then click on the icon for a stylus that appears at the top, and you should be able write, and erase using the icon for an eraser at the top. Otherwise, print a copy of this test on 8.5 by 11 inch letter size paper. If no printer is available, make a hand-written facsimile. Be sure to copy and sign the statement above even if you make a hand-written facsimile. But you do not need to hand-copy this large box of instructions. Do copy each question statement and number however on your facsimile.
- **Show *All* Work** in the space provided. Grading is based on the correctness of the work shown to justify the answers. We can give credit *only* for what you write! *Indicate clearly if you continue a problem on a second page.* There are 8 problems worth 25 points each, for a total of 200 points.
- ***You may use your text book, Zoom recordings of our class meetings, your class notes, and your homework!*** *However, no other sources or communication devices may be used.* ***All work must be your own.*** If you use a calculator, you *must still write out all operations performed* on the calculator. *Do not replace* precise answers, such as $\sqrt{2}$, π, or $\cos\frac{\pi}{7}$ with decimal approximations. *Make all obvious simplifications.* Submit only your own work!
- This is a take-home test on an *honor system.* You may take as much time as you like, but **I must receive your completed test by the end of Tuesday night, April 27.** If you have no tablet device that enables you to write directly on the pdf Exam file, or a device that scans your work directly to a single pdf file, then photograph your pages *in the correct order* with your phone, being sure to *orient all pages the same way,* and save as jpeg, then try this please: put the jpeg files into your computer, highlight the whole group of pictures, right click PRINT and then select PRINT TO PDF. That way I can receive a multi-page PDF file which is possible to grade in a way you will be able to read later. Email that file to me **rich@math.lsu.edu** as soon as you are ready but no later than the end of Tuesday night, April 27. *These instructions express my trust and confidence in your integrity and good character.*

Before you send me your pdf file containing all your pages as one single file, with the problems in the correct order, and please make sure everything is legible. Use a sufficiently dark writing instrument for your test and make sharp, clear images, so I can read them. I simply cannot grade what I cannot read. Thank you for your consideration in this!

Important Note: When you email your completed test back to me, PLEASE put the following in the subject line of your email: 1553_T4_FamilyName_GivenName. This will ensure that your exam is not misplaced into a file of exams from my other class! Thank you.

1. (25) Use *integration by parts* to find $\int x^2 \ln x \, dx$.

2. (25) Use a *trigonometric substitution* to find $\int \frac{\sqrt{x^2 - 9}}{x}\,dx$.

3. (25) Consider the parametric equations $y = te^t$, $x = e^{-t}$, which define a curve C.

a. (10) Find $\dfrac{dy}{dx}$ as a *function of* t.

b. (5) Find the value of t and the coordinates (x, y) at which the *tangent* to the curve C is *horizontal*.

c. (5) Find $\dfrac{d^2y}{dx^2}$ as a *function of* t.

d. (5) Find the interval for t for which the curve C is *concave up*.

4. (25)

a. (10) Sketch *one loop* of the polar graph $r = \cos 2\theta$ containing the point for which $\theta = 0$, and find the range of values of θ corresponding to that loop.

b. (10) *Find* the *area* of the loop found in part (a) above.

c. (5) For the loop identified in part (a), find the two values of θ for which the tangent line will be horizontal. Express θ as either $\sin^{-1}$ or $\cos^{-1}$ of two numbers. Do not replace the exact values by decimal approximations.

5. (25) Test each of the following infinite series for *absolute convergence, conditional convergence, or divergence.*

a. (10) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

b. (10) $\displaystyle\sum_{n=2}^{\infty} \frac{n}{(\ln n)^2}$

c. (5) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\ln n}$

6. (25) Let $f(x) = \dfrac{1}{4-2x}$, $x \neq 2$.

a. (10) Use the *geometric series formula* to express $f(x)$ as $f(x) = \displaystyle\sum_{n=0}^{\infty} a_n x^n$, the sum of a Maclaurin series. That is, find all the coefficients a_n.

b. Find the *radius* R of convergence and the *interval* I of convergence, taking care to say how you *test the endpoints*.

c. (5) Now use *Taylor's coefficient formula* (for a_n) to find $f^{(100)}(0)$ *from the series* already found in part (a).

7. (25) Consider the three points $P(1, 2, 3)$, $Q(6, 5, 4)$, and $R(5, 4, 6)$.

a. (10) Find the vectors $\overrightarrow{PQ}$, $\overrightarrow{PR}$, and $\overrightarrow{N} = \overrightarrow{PQ} \times \overrightarrow{PR}$.

b. (10) Find an equation of the form $ax + by + cz + d = 0$ for the plane containing the points P, Q and R, using the information from part (a).

c. (5) Find parametric equations for the straight line through the point P in the direction of the vector $\overrightarrow{N}$.

8. (25) Consider the space curve C described by the position vector $\vec{r}(t) = \langle 4\cos t, 4\sin t, 3t\rangle$ at time t. Let s denote arc length.

a. (12) Find $\dfrac{d\vec{r}(t)}{dt}$, $\dfrac{ds}{dt}$ and the unit tangent $\vec{T}(t)$ in the direction of increasing t.

b. (8) Find the curvature $\kappa = \left|\dfrac{d\vec{T}}{ds}\right|$ and the unit normal $\vec{n}(t)$ to the curve C.

c. (5) Find the arc length L covered along the curve C from the point $(4, 0, 0)$ to the point $(4, 0, 6\pi)$.

Solutions

1. $\displaystyle\int x^2 \ln x \, dx = \frac{x^3}{3} \ln x - \frac{x^3}{9} + C.$

2. $\displaystyle\int \frac{\sqrt{x^2-9}}{x} \, dx = \sqrt{x^2-9} - 3\sec^{-1}\left(\frac{x}{3}\right) + C.$

3.

a. $\dfrac{dy}{dx} = -(1+t)e^{2t}$

b. $t = -1$ and the coordinates $(x, y) = \left(e, -\frac{1}{e}\right)$

c. $\dfrac{d^2y}{dx^2} = (2t+3)e^{3t}$

d. The curve C is *concave up* for $t > -\frac{3}{2}$. Equivalently: $x < e^{\frac{3}{2}}$.

4.

a. $-\dfrac{\pi}{4} < \theta < \dfrac{\pi}{4}$

b. $A = \dfrac{1}{2}\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos^2 2\theta \, d\theta = \frac{\pi}{8}.$

c. $\theta = \pm \sin^{-1}\left(\dfrac{1}{\sqrt{6}}\right)$

5.

a. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ *converges (absolutely)* by the *integral test.*

b. $\displaystyle\sum_{n=2}^{\infty} \frac{n}{(\ln n)^2}$ *diverges* by the nth *term test.*

c. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\ln n}$ *converges* by the *alternating series test,* but only *conditionally* by *comparison of the absolute series with the harmonic series*, which diverges.

6.

a. $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{1}{2^{n+2}} x^n$, so that $a_n = \dfrac{1}{2^{n+2}}$.

b. (10) $R = 2$, $I = (-2, 2)$, and divergence at each endpoint follows from the nth term test.

c. Since $a_n = \dfrac{f^{(n)}(0)}{n!} = \dfrac{1}{2^{n+2}}$, $f^{(100)}(0) = \dfrac{100!}{2^{102}}$.

7.

a. $\overrightarrow{PQ} = \langle 5, 3, 1 \rangle$, $\overrightarrow{PR} = \langle 4, 2, 3 \rangle$, and $\vec{N} = \overrightarrow{PQ} \times \overrightarrow{PR} = \langle 7, -11, -2 \rangle$.

b. $7x - 11y - 2z + 21 = 0$

c.

$$\begin{aligned} x &= 7t + 1 \\ y &= 2 - 11t \\ z &= 3 - 2t \end{aligned}$$

8.

a. $\dfrac{d\vec{r}(t)}{dt} = \langle -4\sin t, 4\cos t, 3\rangle$, $\dfrac{ds}{dt} = 5$ and the unit tangent $\vec{T}(t) = \frac{1}{5}\langle -4\sin t, 4\cos t, 3\rangle$

b. $\kappa = \left|\dfrac{d\vec{T}}{ds}\right| = \dfrac{4}{25}$ and the unit normal $\vec{n}(t) = \langle -\cos t, -\sin t, 0\rangle$

c. $L = 10\pi$

Class Statistics

Grade	Test#1	Test#2	Test#3	Final Exam	Final Grade
90-100 (A)	15	14	9	10	11
80-89 (B)	4	5	6	7	9
70-79 (C)	3	3	3	11	4
60-69 (D)	0	1	3	2	0
0-59 (F)	0	0	3	0	0
Test Avg	89.5%	88.91%	81.67%	84.55%	87.7%