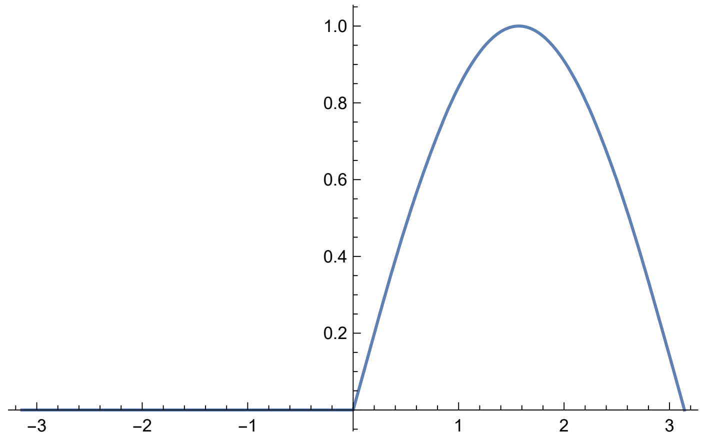

Figure 1: Sum of Corrected Fourier Series

We correct WebAssign's *incorrect* solution to Zill Advanced Engineering Math 12.2 Problem 4. Here we let

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \sin x, & 0 \le x \le \pi \end{cases}$$

and we are asked find the Fourier series of f.

The correct Fourier series is:

$$f(x) = \frac{1}{\pi} + \frac{\sin(x)}{2} + \sum_{n=2}^{\infty} \frac{((-1)^n + 1)\cos(nx)}{\pi\,(1-n^2)}$$

Mathematica plots this infinite series as shown in Figure (1) above. There are two errors in WebAssign's soluiton. First, the constant term was miscalculated because the length of the interval is 2π. The constant term is *always the average value* of the function on the whole interval, which is $\frac{1}{\pi}$ in this example. The second error is that although almost all of the sine coefficients b_n vanish, there is one nonzero sine term in the Fourier series: $\dfrac{\sin(x)}{2}$. In other words, $b_1 = \frac{1}{2}$.

Here I wish to emphasize how one can recognize that the solution demanded by WebAssign is wrong. First, the sum of a pure cosine series must be an *even* fuction. But here f is not even. Thus there had to be at least one missing sine term. The error in the constant term arose because the person calculating the constant term forgot that the domain of f has length 2π although f vanishes on half of that interval. This shows how important it is to be aware of the conceptual aspects of mathematics, especially when dealing with computers.

I will go through the grades assigned and give credit to everyone who was misgraded on this problem.