I have read, understood, and complied with the instructions in the box below. Legible

***Signature* and LSU ID #:** ______________________________

- Download and print a copy of this test on 8.5 by 11 inch letter size paper. If no printer is available, make a hand-written facsimile. Be sure to sign the statement above.
- **Show all work** in the space provided. Grading is based on the correctness of the work shown. We can give credit *only* for what you write! *Indicate clearly if you continue a problem on a second page.*
- ***You may use your text book, Zoom recordings of our class meetings, your class notes, and your WebAssign Homework!*** *However, no other sources or communication devices may be used.* ***All work must be your own.*** If you use a calculator, you *must still write out all operations performed* on the calculator. *Do not replace* precise answers, such as $\sqrt{2}$, π, or $\cos\frac{\pi}{7}$ with decimal approximations. *Make all obvious simplifications.* Submit only your own work!
- This is a take-home test on an *honor system.* You may take as much time as you like, but **I must receive your completed test by email no later than 2:30 PM on Saturday, October 17.** If you have no device that scans your work directly to a single pdf file, then photograph your pages in the correct order with your phone and save as jpeg, then try this please: put the jpeg files into your computer, highlight the whole group of pictures, right click PRINT and then select PRINT TO PDF. That way I can receive a multipage PDF file which is possible to grade in a way you will be able to read later. Email that file to me **rich@math.lsu.edu** as soon as you are ready but no later than Saturday, October 17, at 2:30 PM. *These instructions express my trust and confidence in your integrity and good character.*

Before you send me your pdf file containing all your pages as one single file, please make sure everything is legible. Use a sufficiently dark writing instrument for your test and make sharp, clear images, so I can read them. I simply cannot grade what I cannot read. Thank you for your consideration in this!

1. (30) All solutions to Equation $y'' - xy' + 2y = 0$ below have the form $y = \sum_{k=0}^{\infty} a_k x^k$.

a. (20) Find a *recursion formula* for all a_{k+2} in terms of a_k and/or a_{k+1}.

b. (10) Find the unique solution for which $y(0) = 1$ and $y'(0) = 0$.

2. (40) The differential equation $xy'' + 2y' + 4xy = 0$ has a regular singular point at $x = 0$. The Method of Frobenius assures the existence of at least one solution of the form $y = \sum_{m=0}^{\infty} a_m x^{m+r}$ where $a_0 \neq 0$.

a. (10) Use the coefficient of the lowest power of x to find the *indicial equation* and its roots r_1 and r_2, $r_1 > r_2$, and note that $r_1 - r_2$ *is* an integer.

b. (10) Use the coefficient of the second lowest power of x to determine the value of a_1, regardless of which of the 2 roots for r we use.

c. (10) Find the *recursion relation* that determines a_{m+1} for all $m \geq 1$ in terms of a_{m-1} and r. Don't yet replace r with either of its two roots. What do you learn about all the *odd-indexed* coefficients?

d. (10) *Using the* ***larger*** *root* r_1, and choosing $a_0 = 1$ for convenience, find the *general coefficient* a_k in terms of k. Express the solution y_1 corresponding to r_1 as the sum of a series using the preceding information.

e. (This part is *not required* but is for **10 Bonus Points** on your score.) Now use the smaller root r_2 and choose $a_0 = 1$ to find a second solution y_2, which turns out to be linearly independent of y_1. Write the *general solution*, expressing it as an elementary function.

3. (30) Use the steps (a)-(c) below to solve

$$y'' + 2y' + (e^{2x} - 3)y = 0. \tag{1}$$

a. (10) Define a new *independent* variable $z = e^x$ and express y' and y'' in terms of z and derivatives with respect to z.

b. (10) Introduce a new *dependent* variable $w = e^x y$ and express y and its derivatives with respect to z in terms of w and its derivatives with respect to z.

c. (10) Rewrite Equation (1) in terms of w and z, showing that it is a Bessel equation of some order ν. Then write the general solution to Equation (1) for y in terms of x.