1. p. 12, *Additional Exercise.* Suppose $x = \frac{p}{q} \neq 0$ with p and $q \in \mathbb{N}$ and such that $x^2 \in \mathbb{N}$. *Fix* such an x for the remainder of this exercise. Prove by the following steps that $x \in \mathbb{N}$. (This can be expressed as follows: A natural number that is not a perfect square has no rational square root.)

   (a) Show that the set $S = \{a + bx \mid a, b \in \mathbb{Z}\}$ is closed under both addition and multiplication, where x is as given and fixed above.

   (b) Show that if $s \in S \setminus \{0\}$ then $|s| \geq \frac{1}{q}$.

   (c) Show that if $x \notin \mathbb{N}$ and $\lfloor x \rfloor$ is the *greatest integer* in x, then $(x - \lfloor x \rfloor)^k \to 0$ as $k \to \infty$ and is a sequence of nonzero numbers in S. Show that this is a contradiction, so that $x \in \mathbb{N}$.