Comments concerning $P[X < t]$ and $P[X < t]$

The DISTRIBUTION FUNCTION F_X for a random variable X is define by

$$F_X(t) = P[X \leq t]$$

Observe that we have here $\leq$, not $<$.
The probability of the event $[X < t]$ can be obtained by as a limit:

$$P[X < t] = \lim_{x \to t-} P[X < t] = \lim_{x \to t-} F_X(t)$$

If the function F_X happens to be continuous at t then, and only then, $P[X < t]$ is equal to $F_X(t)$.

Example.
Consider the distribution function given by

$$F_X(x) = \begin{cases} 0 & \text{if } x < -1 \\ \frac{1}{3} + \frac{2}{3}(x+1)^2 & \text{if } -1 \leq x \leq 0 \end{cases}$$

The values of $F_X(x)$ for $x > 0$ are not stated. This is because

$$F_X(0) = \frac{1}{3} + \frac{2}{3} = 1$$

which means $P[X \leq 0] = 1$, and so $F_X(x) = 1$ for all $x \geq 0$.
Observe also that

$$P[X \leq -1] = F_X(-1) = \frac{1}{3} + 0 = \frac{1}{3}$$

while, on the other hand, $P[X < x] = F_X(x) = 0$ for any $x < 0$. This implies that

$$P[X = -1] = \frac{1}{3}$$

What is happening here is that the function F_X has a discontinuity at -1.
Let us calculate $P[|X - \frac{1}{3}| < 1]$:

$$\begin{aligned} P\left[|X - \frac{1}{3}| < 1\right] &= P\left[-\frac{2}{3} < X < \frac{4}{3}\right] \\ &= P\left[X < \frac{4}{3}\right] - P\left[X \leq -\frac{2}{3}\right] \\ &= 1 - F_X\left(-\frac{2}{3}\right) = 1 - \left[\frac{1}{3} + \frac{2}{3}\frac{1}{3^2}\right] \end{aligned}$$