Math 4200 Section 2/ Fall 2002 Test 3

Name:_____

Please provide full explanations with your answers. No credit will be given for an answer which is not explained. Your work must be presented in a clean and formal way.

1. Suppose H and K are finite subgroups of a group G such that $|H|$ and $|K|$ are coprime. Prove that $H \cap K = \{e\}$, where e is the identity element in G. (10pts)

2. Write out all the six elements of S_3 and then write out all possible subgroups of S_3. (10pts)

3. Recall that for any integer m, we have the quotient $\mathbf{Z}_m = \mathbf{Z}/m\mathbf{Z}$ which is a group under addition. Elements of $\mathbf{Z}_m$ are the cosets $a+m\mathbf{Z}$ with a running over all integers. On $\mathbf{Z}_6$ consider the mapping given by

$$f : \mathbf{Z}_6 \to \mathbf{Z}_6 : x + 6\mathbf{Z} \mapsto 3x + 6\mathbf{Z}$$

This is well-defined and is a homomorphism. Write out the kernel of f and the image of f. (10pts)

4. Let G be a group, x an element of G, and let C_x be the set of all $y \in G$ which commute with x, i.e.

$$C_x = \{y \in G : yx = xy\}$$

Verify that C_x is a subgroup of G. (10pts)