Partial Differential Equations

Math 7380-3, Fall 2009
Louisiana State University
Tuesday and Thursday, from 1:40 to 3:00 PM
Room 130 of Lockett Hall

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Office hours: Monday 9:00–11:00 and Wednesday 1:00–3:00 or by appointment
Course web site: `http://www.math.lsu.edu/~shipman/courses_09B-7380`

Course Synopsis:
The aims of the course are these:

- To provide an overview of the **diversity** of mathematics and applications subsumed by or connected to partial differential equations.
- To delve into the theory of some **specific topics**.
- To impart a necessary **intuition** for PDEs, specifically for those of mathematical physics. This entails delving into the details of specific examples.
- To learn **techniques and theory** from hard analysis and abstract functional analysis that are necessary tools for analyzing PDEs. Most of the time all the details of the theory we use can be provided. At times we will have to use a theorem without proof, such as the spectral theorem; in these cases, we will understand the significance and application of the theory and give references to its development.

Here is a list of topics that I would like to discuss, not necessarily in the order given. We will definitely do the first eight topics in some capacity.

1. Basic linear equations and separation of variables: the diffusion equation, the wave equation, and the Laplace and Poisson equations. Fundamental solutions.
2. Elliptic equations. Weak formulation of PDEs and the associated abstract functional analysis; applications to materials science.
3. Unitary groups, and the role of self-adjoint extensions of symmetric operators in boundary-value problems.
4. Fourier transform methods.

5. Waves. Nonlinear and linear waves, dispersion relations, hyperbolic systems and shocks; the Korteweg-deVries equation.

6. Asymptotic analysis: the method of stationary phase and WKB theory; application to the vanishing-dispersion limit of the KdV equation.

7. The method of characteristics for first-order equations.

8. Calculus of variations, the Hamilton-Jacobi equation, the iconal equation in geometric optics.

9. Boundary-integral equations and their connection to complex variables and wave scattering.

10. Special relativity: differential geometry and the special role of the wave equation.

11. Quantization and the Schrödinger equation.

Literature:

We will not use any one source as a text book. My lectures will draw from my notes and several references, including the books of P. R. Garabedian, L. C. Evans, E. C. Titchmarsh, G. B. Whitham, F. Treves, F. John, R. Haberman, and J.-C. Nédélec and the functional analysis books of M. Reed and B. Simon.

I will place a bibliography of relevant works on the web site for the course. I will also make available, through links from the bibliography, PDF files of excerpts from the literature.

Assignments:

I will assign problems regularly. At the end of the course, I will give a longer and somewhat comprehensive set of problems that will serve as an out-of-class final exam.

Students may discuss problems with each other and other people (including me, of course) and consult other literature; in fact students are encouraged to search the literature and discuss ideas. However, all work that is turned in must ultimately be that of the submitter alone. If a student receives aid on an assigned problem from discussions with people or other sources, he or she must begin from scratch in writing the solution so that the result is the product of his or her own understanding alone.

Evaluation:

Evaluation of performance in the course is based on performance on the regular assignments and the out-of-class final exam, according the the following weighting:

Regular assignments: 75%
Final exam: 25%