Abstract Degenerate Cauchy Problems in Locally Convex Spaces [*]

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This paper is concerned with the abstract degenerate Cauchy problem (DCP): $\frac{d}{dt}Bu(t) = Au(t)$ $(t \geq 0)$, $Bu(0) = Bu_0$, where A and B are closed linear operators in a sequentially complete locally convex space. A C-propagation family for (DCP) is introduced, leading to a general C-wellposedness result about (DCP). Moreover, conditions are given ensuring the existence of C-propagation families for those (DCP) with differential operators, on various function spaces with Fréchet topologies, as coefficient operators. These results are new even in the case of Banach spaces.

0. INTRODUCTION

Let E be a sequentially complete locally convex space topologized by $\mathbf{\Gamma}$ of seminorms, and let A, B be closed linear operators in E. We consider the following abstract degenerate Cauchy problem

$$\begin{cases} \dfrac{d}{dt}Bu(t) = Au(t), & t \geq 0, \\ Bu(0) = Bu_0. \end{cases} \qquad (DCP)$$

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[*] This work was supported partly by the K. C. Wong Education Foundation, the Key Project of the Chinese Academy of Sciences and the National NSF of China

The (DCP) has been studied by many researchers in the setting of Banach spaces (see, e.g., [1, 3, 4, 7, 8, 10, 15] and references therein). In this paper, we introduce a C-propagation family $\{W(t)\}_{t\geq 0}$ for (DCP) in a locally convex space, which is a generalization of the C-regularized semigroup and so the classical strongly continuous semigroup(cf., e.g., [5 - 7, 9, 11, 14]). In terms of $\{W(t)\}_{t\geq 0}$, we derive a general C-wellposedness result regarding (DCP) (Theorem 1.6). On the other hand, we pay attention to those (DCP) with differential operators, on various function spaces endowed with Fréchet topologies, as coefficient operators, which can not be treated by the known theory about (DCP). We obtain