

STUDENT NAME:

Calculus 1550, section 20. Third test. Thursday, October 30, 2003.

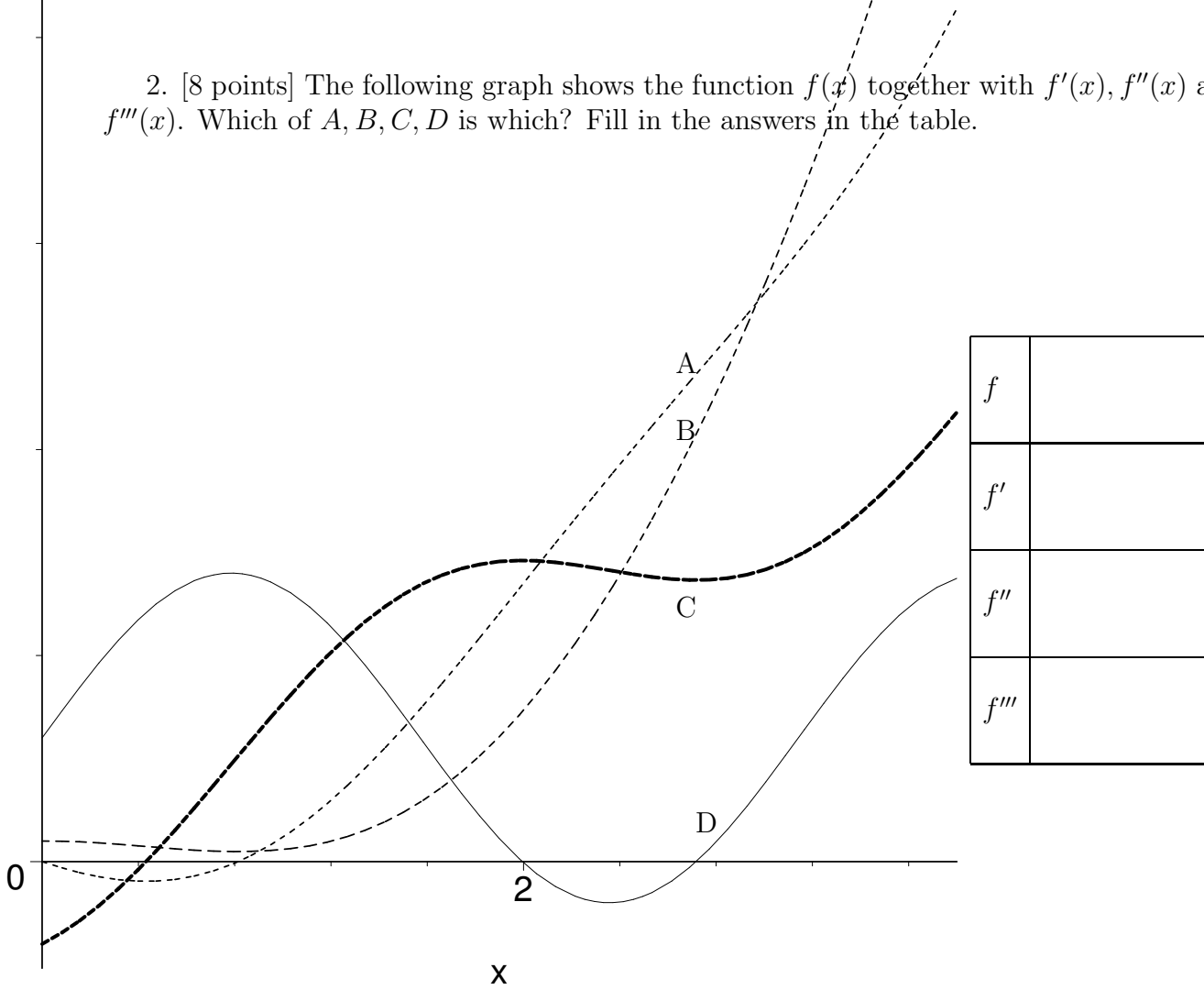
This test paper has 6 pages. Points per question are given in square brackets.

1. Use implicit differentiation to find $\frac{dy}{dx}$.

i. [5 points] $x^4 + y^2 + 1 = \sin(x)$

ii. [5 points] $(x^3 + y)^{-9} = 9x$

2. [8 points] The following graph shows the function $f(x)$ together with $f'(x)$, $f''(x)$ and $f'''(x)$. Which of A, B, C, D is which? Fill in the answers in the table.



3.i. [9 points] Find the first, second and third derivatives of $f(x) = \sin(x) \cos(x)$.

3.ii. [2 point] What is $\frac{d^6 \sin(x) \cos(x)}{dx^6}$?

4. Differentiate the following functions

i. [5 points] $f(x) = \ln(\cos^2(x))$

ii. [6 points] $f(x) = \cos(x)^x$

5. [10 points] Sketch the graph of a function on the interval $[-5, 5]$ which has on this interval:

2 local minimum

1 local maximum

no absolute maximum

6 critical numbers

6. [10 points] Find the absolute maximum and absolute minimum values of the function

$$f(x) = 3x^4 - 10x^3 + 6x^2$$

on the interval $[0, 1]$.

7. [10 points] Let

$$f(x) = x^2 + 3x + 1.$$

Find a number c in the interval $[1, 2]$ which satisfies the conclusion of the mean value theorem for the function $f(x)$ on the interval $[1, 2]$.

3. [10 points] Two ships start from the same point at 12 noon. One travels north at 5 miles/hour and the other travels east at 12 miles/hour. How fast are they going away from each other at 1pm? Show your working, including a labeled diagram to explain your notation.

9. [10 points] Use L'hospital's rule to find the following limits.

1. $\lim_{x \rightarrow \infty} \frac{3x^5 + 1}{2x^5 - x + 1}$.

3. $\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 9} - 3}{x^2}$.

10. [10 points] On which intervals is the function

$$f(x) = x^5 - 5x + 1$$

increasing and decreasing?