Test solutions, with approximate grading scheme

STUDENT NAME:

Calculus 1550, section 5. Second test. Tuesday, October 3, 2004.

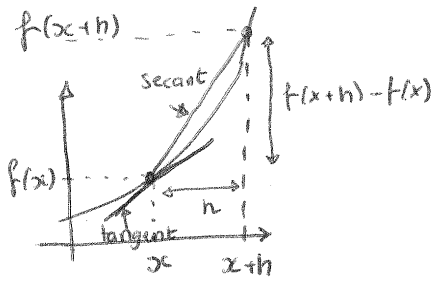
This test paper has 4 pages. Points per question are given in square brackets. The total is 50 points.

Put your name on this sheet and your initials on each page.

Q1. [5 points]
What is the definition of the derivative of a function $f(x)$ in terms of limits?

h & x mixed up, but otherwise OK - 4pts.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



missing limit sign - 3 pts., ∞ instead of 0 - (3)

Q2. [5 points]
Give an example of a function $f(x)$ which is continuous at $x = 4$, but such that $f'(4)$ is not defined. Give an equation for $f(x)$, and sketch a graph of $f(x)$.

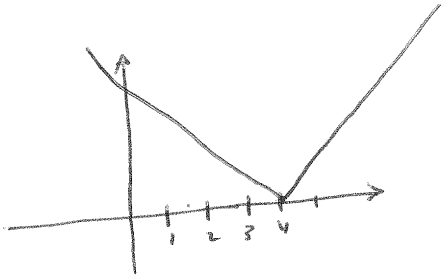
(5) - everything correct

(3 or 4, depending on mistake)
- correct graph, but no function or incorrect function
- correct function, no graph, or incorrect graph

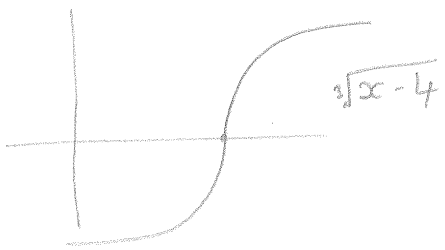
(2) - a function which is not differentiable, but also not continuous at 4 with graph & function matching

(1) - above, but graph & function do not match

example: $f(x) = |x-4|$ graph:



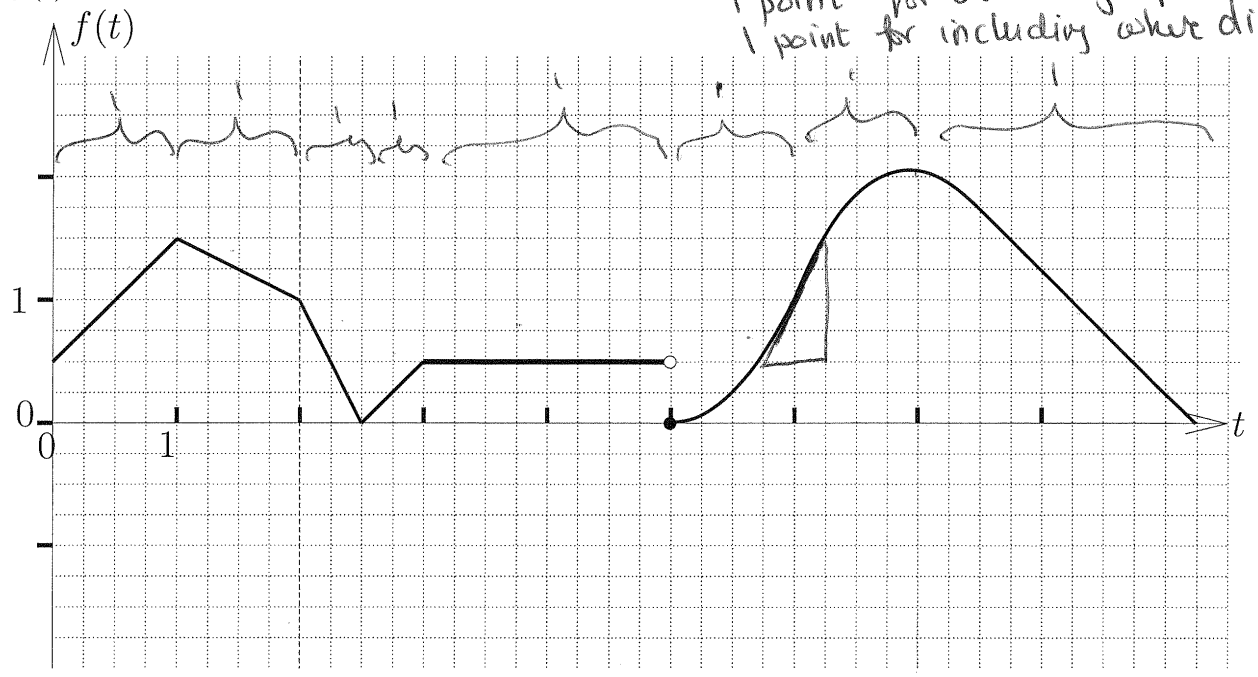
another example:



Q3. [10 points]

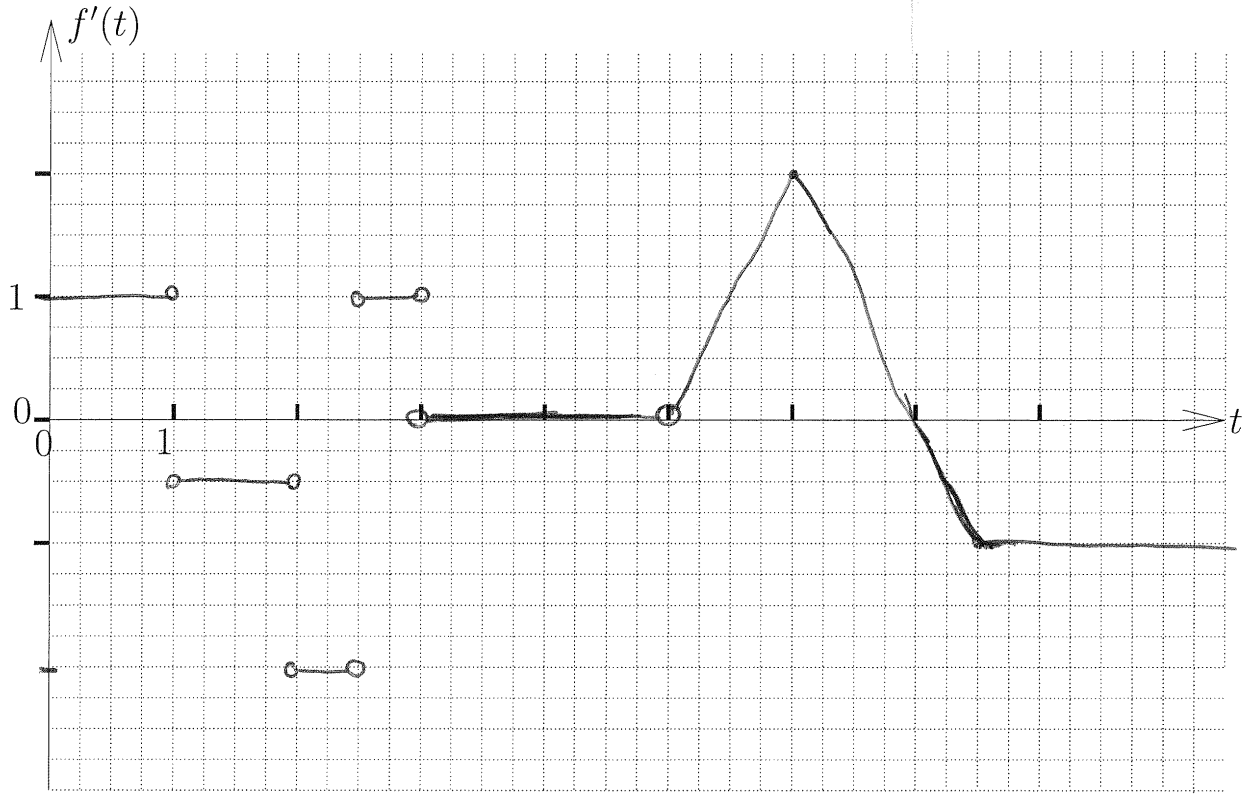
1 point for each section marked
1 point for overall graph
1 point for including where discontinuities are.

$f(t)$ is a function with a graph as follows:



Sketch the graph of $f'(t)$ below. Make sure your graph shows the following features:

i Where $f'(t)$ is zero, positive and negative.
ii Where $f'(t)$ is decreasing or increasing.
iii Where $f'(t)$ is undefined, infinite (by drawing an asymptote), or has a discontinuity.
iv Exact values of $f'(t)$ when $f'(t)$ is constant on some interval.



Q6.

Find the derivatives of the following functions [5 points each].

i. $f(x) = e^x \cos(x)$

correct derivatives of e^x — (2) (1)
& $\cos(x)$ — (2) (1)

Knowledge of product rule — (3) (1)
putting these together correctly — (3) (2)

$$f'(x) = e^x(-\sin(x)) + e^x\cos(x)$$
$$= e^x(\cos(x) - \sin(x))$$

ii. $f(x) = \dfrac{\sin(x)}{x^2}$

correct derivative of $\sin(x)$ (1)
—11— x^2 (1)
Knowledge of quotient rule (1)
putting these together correctly (2)

$$f'(x) = \frac{x^2\cos(x) - 2x\sin(x)}{x^4}$$

$$= \frac{\cos(x)}{x^2} - \frac{2}{x^3}\sin(x)$$

Q4. [10 points]
For which values of x does the graph of the following function have a horizontal tangent?

$$f(x) = 2x^3 + 3x^2 - 72x + 7$$

find derivative correctly -(4)
factor correctly -(3)
set factors to zero & get correct solutions -(3)

$$f'(x) = 6x^2 + 6x - 72 = 6(x^2 + x - 12)$$

if $f'(x) = 0$, then $6(x^2 + x - 12) = 6(x + 4)(x - 3)$

so $x = +3$ or $x = -4$

so $f(x)$ has a horizontal tangent at $x = -3$ and $x = 4$

Q5. [10 points.] Find the equation for the tangent of the following function at $(1, 4)$

$$f(x) = x^3 + 2x + 1$$

find derivative — (3)
evaluate derivative at $x = 1$ — (2)
recognize that this is the slope -(1)
find the equation of the line, with correct y intercept - (4)

$$f'(x) = 3x^2 + 2x$$

at $x = 1$, $f'(1) = 3 + 2 = 5$

so, tangent at (1,4) satisfies $\frac{y - 4}{x - 1} = 5 \Rightarrow y - 4 = 5x - 5$

$$\boxed{y = 5x - 1}$$

equation of tangent when $x = 1$