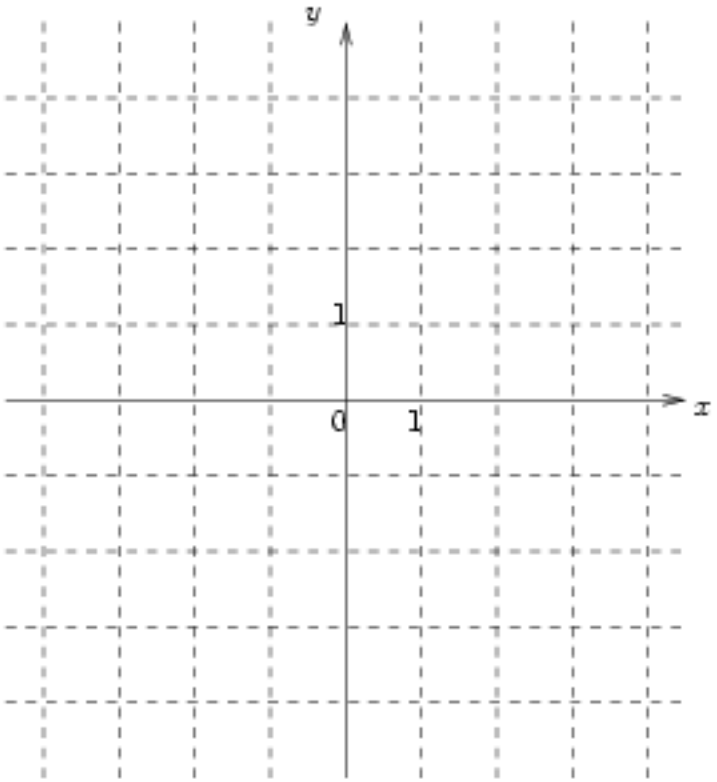
Math 1550 section 5 – Sample Exam. Exam is on Tuesday 7 December 2004, 10am – noon
There will be more space on the final exam, and fewer questions.
Many of these questions are taken from previous exams, so solutions can be found on web pages.

Q1.

In the grid below, sketch a graph with domain $[-4,4]$ which satisfies:

0. $f(x)$ is continuous on $[-4,-2)\cup(-2,2)\cup(2,4]$
i. $\lim_{x\to -2^-} f(x) = 2$
ii. $\lim_{x\to -2^+} f(x) = -3$
iii. $f(-2) = -1$
iv. $\lim_{x\to 2} f(x) = -1$
v. $f(2) = 4$
vi. f is increasing on $(-4,-2)$
vii. f is decreasing on $(2,4)$
viii. f is concave up on $(-2,0)$
ix. f is concave down on $(0,2)$
x. f has a local maximum when $x = 1$.



Q2.

What is the absolute minimum value of the function

$$f(x) = x^3 - 6x^2 + 9x + 1$$

on the interval $[0,2]$?

Q3.

Find the derivatives of the following functions.
i. $f(x) = x^7 + \tan(x) - 2^x$
ii. $f(x) = \sin(\cos(x))$
iii. $f(x) = x^{e^x}$

Q4.

Use implicit differentiation to find $\frac{dy}{dx}$.
i. $\cos(x)\sin(y) = 1$
ii. $\ln(y^3) = x^2$

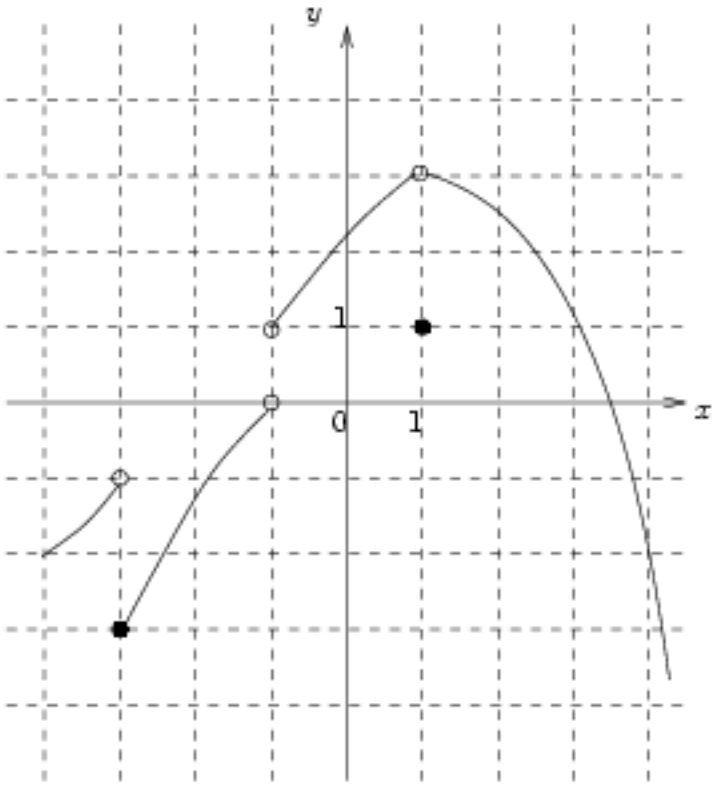
Q5.

Find the equation for the tangent to $y = x^2 - 3x - 1$ at $x = 2$.

Q6.

If $f(x)$ is a function with $f(3) = 2$ and $f'(3) = -2$,
then if $g(x) = x^2 f(x)$, $h(x) = \dfrac{e^x}{f(x)}$, $k(x) = f(x^2)$ and $m(x) = (f(x))^2$,
what are $g'(3)$, $h'(3)$, $k'(3)$, and $m'(3)$?

Q7. f is a function with graph as given. For each quantity below, either write its value, or "Does not exist", if it does not exist.



$\lim_{x\to -3} f(x)$	
$\lim_{x\to -1^-} f(x)$	
$f(1)$	
$\lim_{x\to 1^+} f(x)$	
$\lim_{x\to 1} f(x)$	
$\lim_{x\to 4} f(x)$	

Q8. What is the volume of the solid obtained by rotating the area between the curve $y = e^x$ and the x axis, from $x = 0$ to $x = 1$, about the x-axis?
(Leave your answer as an expresion in terms of e and π.)

Q9. what is the average value of the function $f(x) = x^4\sqrt{81 - x^5}$ on the interval $[0, 2]$?

Q10. Compute the finite area enclosed between the following two curves:

$$y = f(x) := x^4 + x^3 + x^2 - x + 1$$

$$y = g(x) := x^4 - x^3 + x^2 + x + 1$$

Q11. A farmer has 100m of fencing, and wants to enclose the largest possible rectangular area, with fencing on all four sides, which must be divided into two regions, with another length of fence. What is the maximum possible area?

Q12. Let $f(x)$ be the function

$$f(x) = \frac{1 + x^3}{1 - x^3}.$$

i. What is the domain of f? _____

ii. Is f odd, even, or periodic, or has no symmetries? _____

iii. What are the x and y intercepts for $f(x)$?

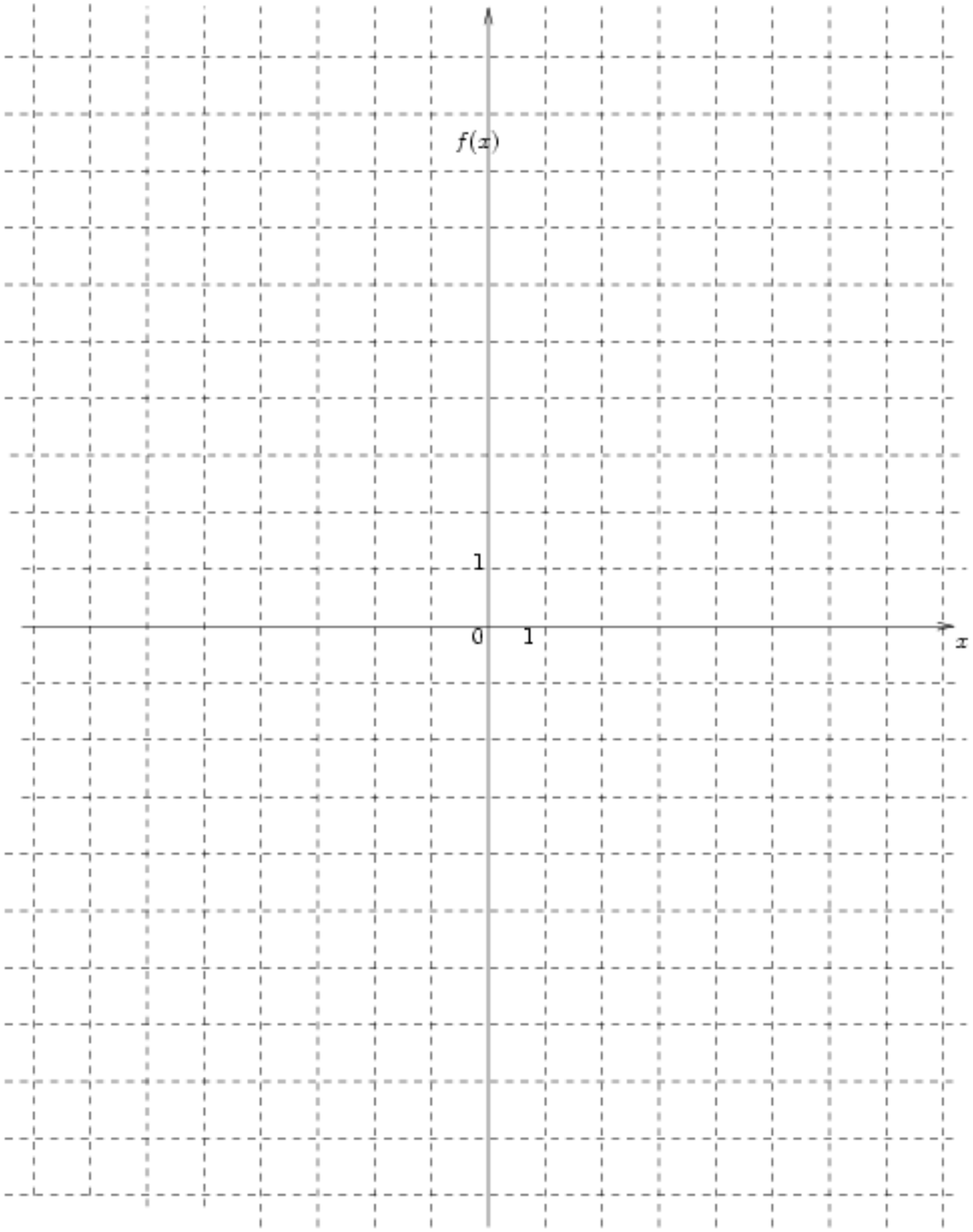
iv. Does $f(x)$ have vertical asymptotes, and if so, what are they?

v. Does $f(x)$ have horizontal asymptotes, and if so, what are they? _____

vi. What are the critical numbers of $f(x)$?
What is the value of $f(x)$ at the critical numbers in the domain of $f(x)$?

vii. On which intervals is $f(x)$ increasing? _____

viii. Sketch a graph of $f(x)$, showing the above features.

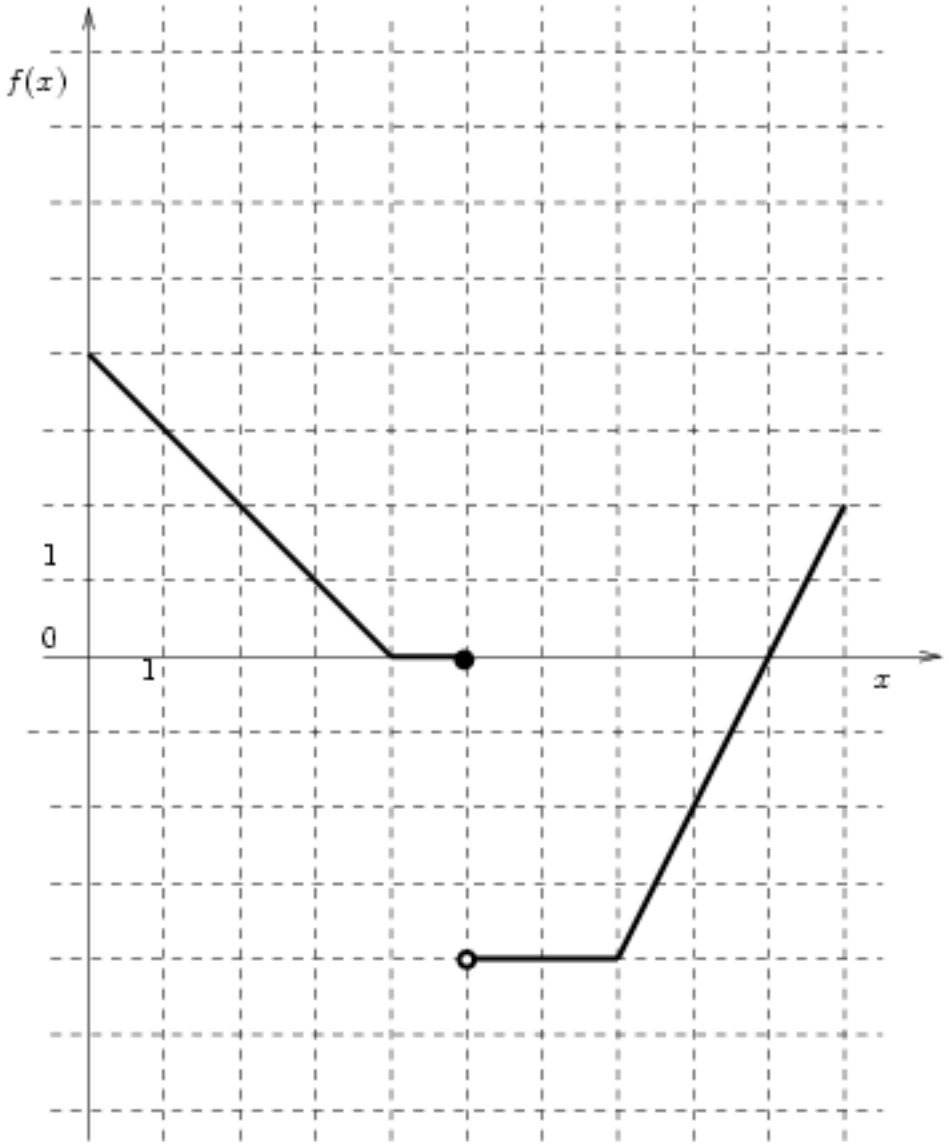


Q13. If a 10ft long ladder slides down a wall, with the end touching the wall sliding down at 2ft/s, what is the speed of the bottom end when it's 3ft from the wall?

Q14. Below the graph of a function $f(x)$ is sketched.

i. On the same grid, Sketch a graph of the function

$$g(x) = \int_0^x f(t)dt$$



ii. What is the maximum value of $g(x)$ on $[0, 10]$?

iii. What is the minimum value of $g(x)$ on $[0, 10]$?

iv. On what intervals is $g(x)$ concave up?

v. What are the values of $g(2)$, $g'(2)$, $g''(2)$ and $g'''(2)$?

iv. Suppose

$$\int_a^x f(t)dt = g(x) - 3$$

What is a possible value of a?

Q15. Find the following definite and indefinite integrals. Show your working and reasoning. In the indefinite cases, give the most general form.

i. $\int \frac{x^2+1}{x}dx$. ii. $\int_0^2 \frac{x}{x^2+1}dx$. iii. $\int x^2(8-x^3)^9 dx$ iv. $\int_{-1}^{1} x^4 \sin(x^5)dx$ v. $\int_1^1 \sin^9(x)dx$