

Math1550, section 20. Calculus. FINAL EXAM, Thursday 11 December, 2003,
Locket 138, 12:30—2:30 pm

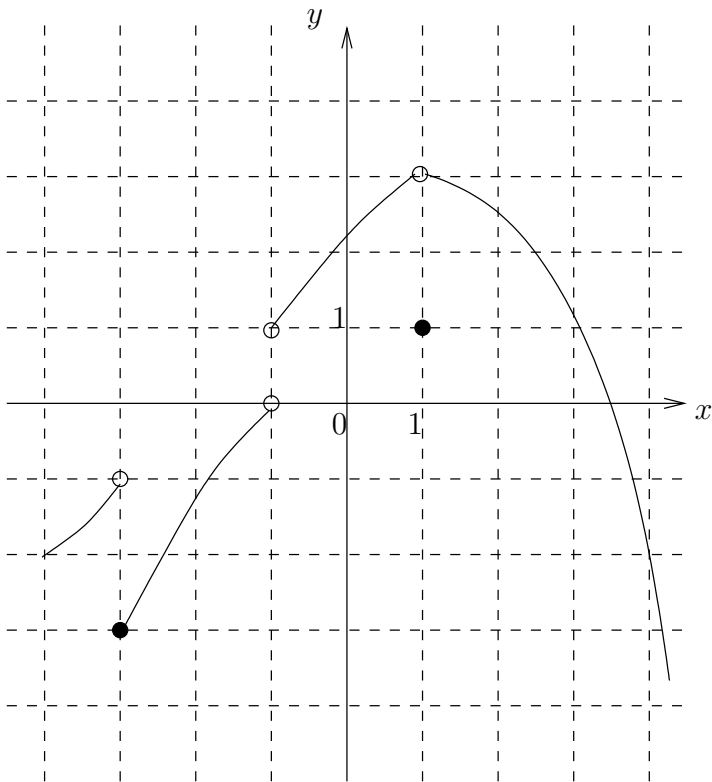
STUDENT NAME:

This exam has 16 pages, and 10 questions (some questions have more than one part).
There are 10 points per question.

Please put your initials at the top of each page.

Please show your working.

1. [10 points] f is a function with graph as given. For each quantity below, either write its value, or “Does not exist”, if it does not exist.

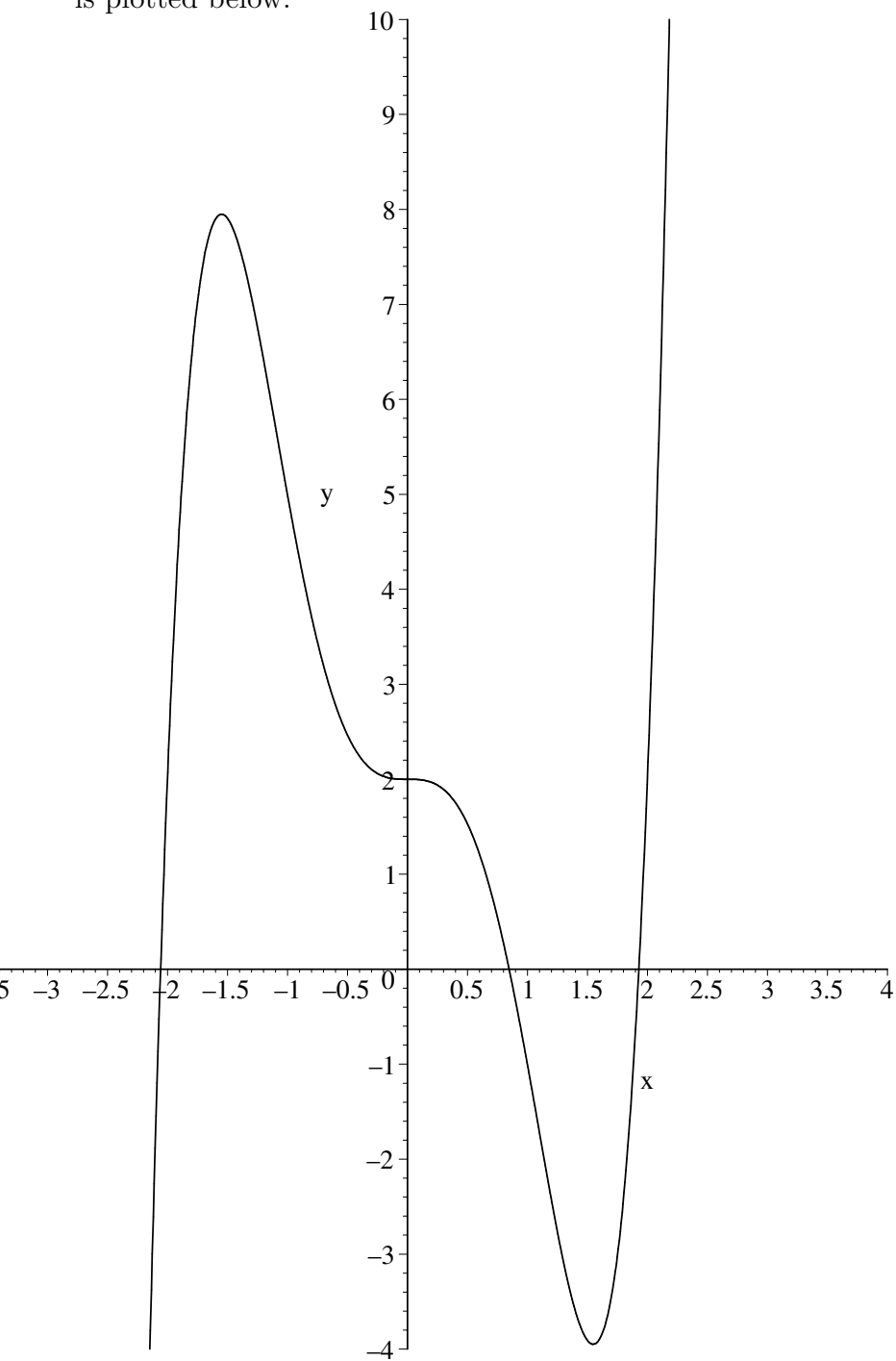


$\lim_{x \rightarrow -3} f(x)$	
$\lim_{x \rightarrow 1^-} f(x)$	
$f(1)$	
$\lim_{x \rightarrow 1^+} f(x)$	
$\lim_{x \rightarrow 1} f(x)$	
$\lim_{x \rightarrow 4} f(x)$	

2. [10 points] The graph of a function

$$y = f(x) = x^5 - 4x^3 + 2$$

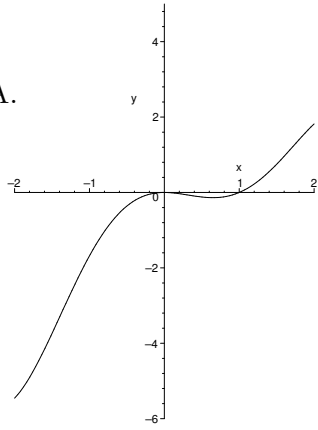
is plotted below.



i. Write an equation for the tangent to the curve $y = f(x)$ at $x = 2$, and sketch this tangent on the diagram.

ii. Find an approximate solution to $f(x) = 0$, by using Newton's method, starting with an estimate $x_1 = 2$, and computing x_2 .

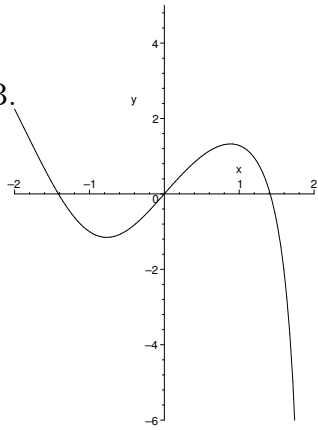
A.



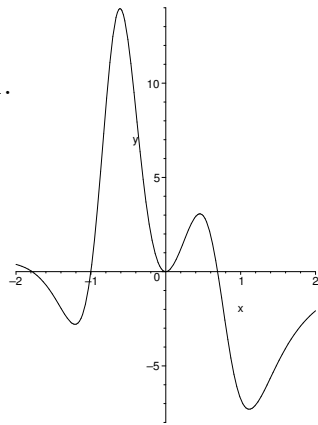
3. [10 points]
 Fill in the table to indicate which of 1–6 is the derivative of each of A–D.

A	
B	
C	
D	

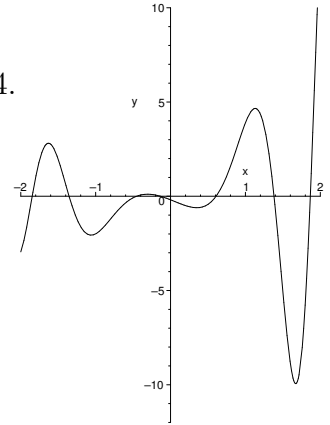
B.



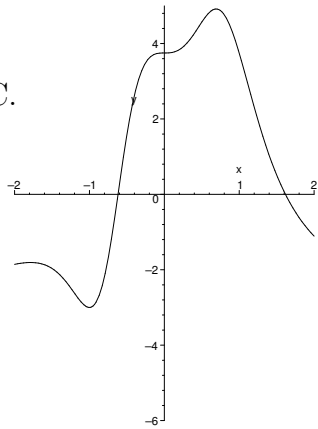
1.



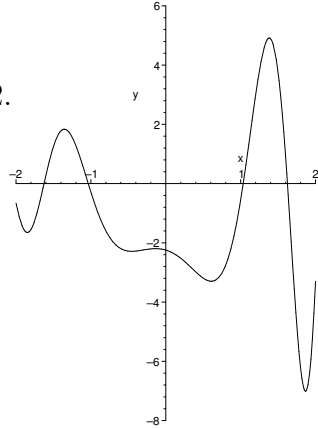
4.



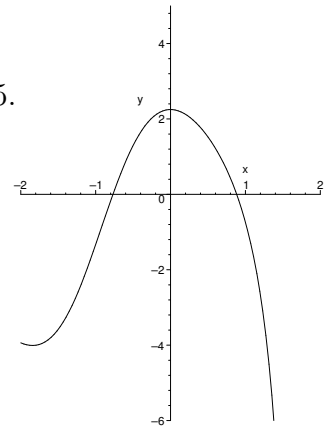
C.



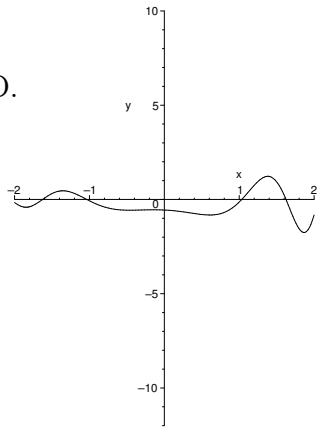
2.



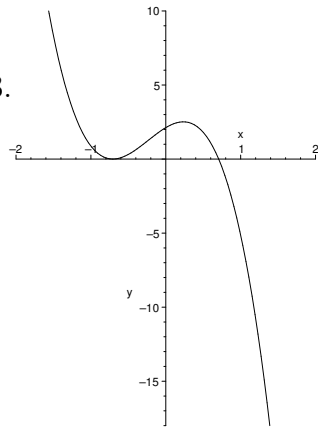
5.



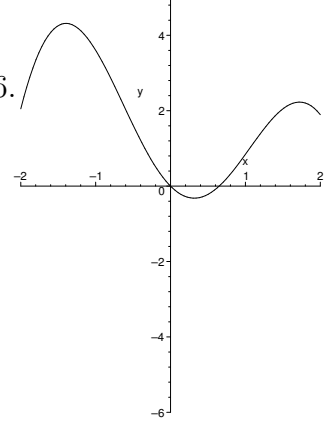
D.



3.



6.



4. [10 points] Find the absolute maximum and minimum values of the following function on the interval $[-1, 1]$

$$f(x) = x^4 - 8x^3 - 14x^2 + 8.$$

5. [10 points] Find the derivatives of the following functions.

i. $f(x) = 4x^5 - \frac{2}{x^3} + \sqrt[3]{x}$.

ii. $f(x) = x^2 \sin(x)$

iii. $f(x) = (x + 1)^{(x-1)}$

6. [10 points] Use implicit differentiation to find $\frac{dy}{dx}$.

i. $x^2 + xy = y^3$

ii. $x \tan(y) = e^x$

7. [10 points] Let $f(x)$ be the function

$$f(x) = \frac{e^{1/x}}{x-2}.$$

i. What is the domain of f ? _____

ii. Is f odd, even, or periodic, or has no symmetries? _____

iii. Compute $\lim_{x \rightarrow 0^+} f(x)$. _____ (Hint: what is $\lim_{z \rightarrow +\infty} e^z$?)

iv. Compute $\lim_{x \rightarrow 0^-} f(x)$. _____ (Hint: what is $\lim_{z \rightarrow -\infty} e^z$?)

v. Compute $\lim_{x \rightarrow +\infty} f(x)$. _____

vi Compute $\lim_{x \rightarrow -\infty} f(x)$. _____

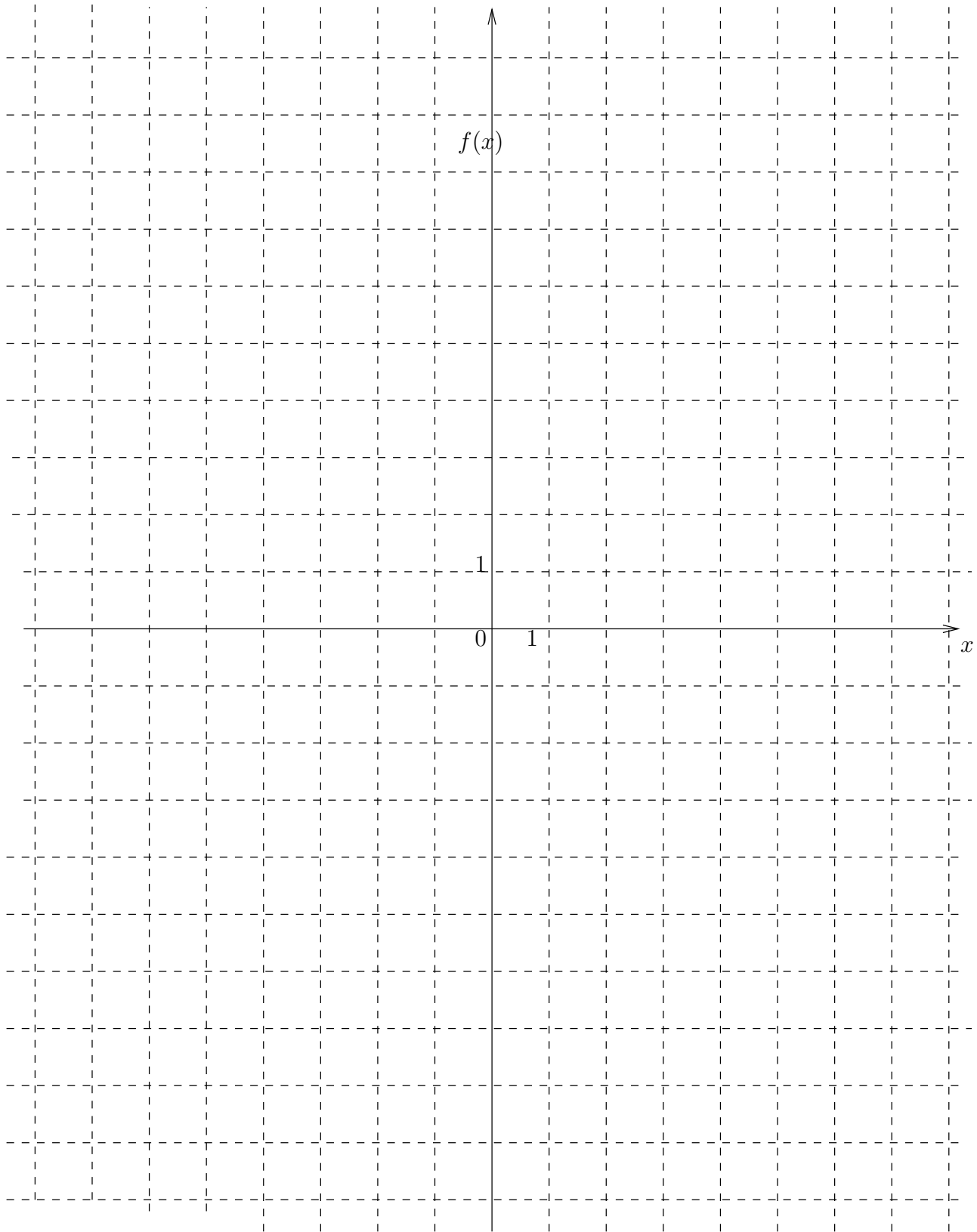
vii. Does f have horizontal or vertical asymptotes, and if so, what are they? _____

v. What are the critical numbers of $f(x)$? _____

vi. On which intervals is $f(x)$ increasing? _____

vii. What is the value of $f(x)$ at local maxima and minima? _____

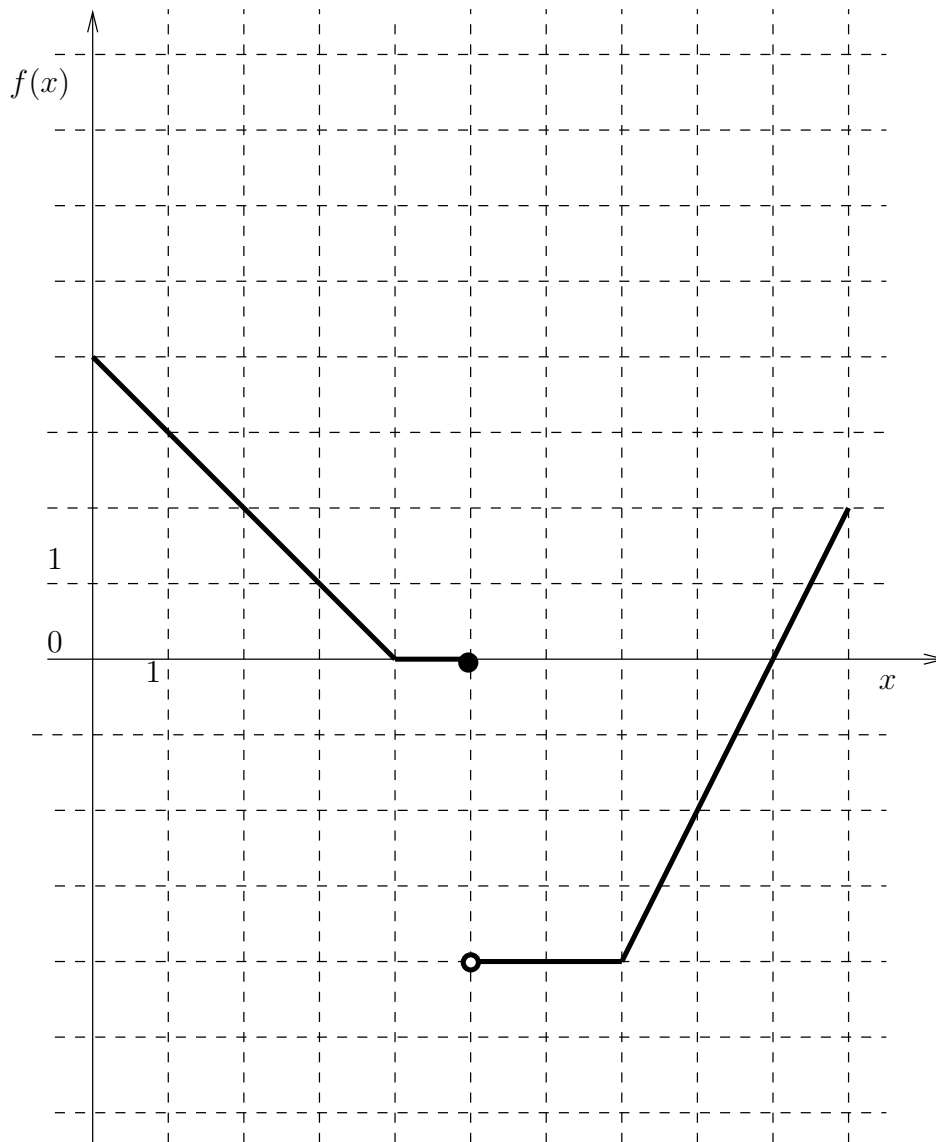
viii. Sketch a graph of $f(x)$, showing the above features.



8. [10 points] Below the graph of a function $f(x)$ is sketched.

i. On the same grid, Sketch a graph of the function

$$g(x) = \int_0^x f(t) dt$$



ii. What is the maximum value of $g(x)$ on $[0, 10]$?

iii. What is the minimum value of $g(x)$ on $[0, 10]$?

iv. Suppose

$$\int_a^x f(t) dt = g(x) + 3$$

What is a possible value of a ?

9A. [5 points] Find the following integrals. Show your working and reasoning.

i. $\int_0^2 (x^3 - 3x^2 + 2x)dx$

ii. $\int_{-1}^1 x^3 \sin(x^2)dx$. (Hint: sketch a graph of this function)

9B. [5 points] what is the average value of the function $f(x) = x^4\sqrt{81 - x^5}$ on the interval $[0, 2]$?

10. [10 points] Compute the finite area enclosed between the following two curves:

$$y = f(x) := x^4 + x^3 + x^2 - x + 1$$

$$y = g(x) := x^4 - x^3 + x^2 + x + 1$$