

Practice questions for preparation for the final exam:

- Sketch the curve $y = (x - 1)(x - 2)$.
 - Sketch the tangent to this curve at $x = 2$, and find the equation of this tangent.
 - Draw a secant to this curve, with slope 2, and passing through the point with $x = 2$. What is the equation for this secant? Where else does this secant cross the curve?
- Find the following limits: i. $\lim_{x \rightarrow 0^+} \frac{|x^2 + x|}{x - 1}$; ii. $\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 1} - 1}{x}$; iii. $\lim_{x \rightarrow \infty} \frac{e^{4x} - x}{e^{5x} + x}$; iv. $\lim_{x \rightarrow 0} x^x$.
- Find an interval of length 1 containing a solution to $2^{-x} = x$.
- Figure A below shows a function $f(x)$, and $f'(x)$, $f''(x)$ and $f'''(x)$ — which is which?
- Figures B1, B2, B3, B4 below show $g(x)$, $g'(x)$, $h(x)$ and $h'(x)$ — which is which?
- Sketch a curve with: a horizontal asymptote; 1 vertical asymptote; 2 discontinuities; 3 places where the derivative is discontinuous; 1 local maximum; 2 local minimums.
 - Sketch the derivative of the function you have just sketched.
- Find the derivatives of the following functions: i. $\sin(x^2 + 1)$; ii. $\tan^3(x)$; iii. $e^{\tan(x)}$; iv. $\tan(x)^e$; v. $\ln(\sin(x))$; vi. $\ln(x) \cos(x)$; vii. $\sin^{-1}(x + 1)$; viii. $\sqrt{(x + 2)/(\sin(x) + 1)}$.
- Find dy/dx in the following cases: i. $x^3 + xy = \sin(x)$; ii. $(x + y)^{10} = \tan(y)$; iii. $x = ye^y$.
- What is the maximum area of a rectangle drawn inside a circle of radius 1?
- If a 6ft tall man walks away from a 12ft lamppost at 5ft/s, how fast is his shadow growing after 4 seconds? (Assume he starts at the lamppost at time $t = 0$ s.)
- Find some $c \in [0, 1]$ satisfying the mean value theorem for $f(x) = (x + 2)/(x - 2)$ on this interval.
- Find the domain, intercepts, symmetries, asymptotes of i) $f(x) = x \sin(x^2)$; find also the critical values, intervals of increasing, decreasing, concave up and concave down, for ii) $f(x) = (x^2 + 4)/(x^2 - 4)$. Sketch both curves.
- Use Newton's method to find a solution to $x + \sin(x) = 2$, correct to 2 decimal places.
- Estimate the value of $\int_0^1 x^2 e^x dx$ by dividing $[0, 1]$ into 4 subintervals, and using the mid-point rule.
- Find a function $f(x)$ with $f'(x) = 2x - 4$ and $f(1) = 0$.
- Compute the following integrals: i. $\int_{-1}^1 (3x^5 + x^3 + x) dx$; ii. $\int_{-1}^1 (4x^6 + x^2 + 1) dx$; iii. $\int_1^2 \frac{3x^2 + 2x}{x^3 + x^2 + 1} dx$; iv. $\int_{-\pi}^{\pi} x^2 \cos(x^3) dx$; v. $\int_0^2 x^2 \sqrt{x^3 + 1} dx$; vi. $\int_1^2 \frac{\sin(1/x)}{x^2} dx$.
- Let $g(x) = x$ on $[0, 1]$ and $[2, 4]$; $g(x) = -x$ on $[1, 2]$ and $[4, 6]$, and $g(x) = 0$ elsewhere.
 - Sketch a graph of $g(x)$.
 - Sketch a graph of $g'(x)$.
 - Sketch a graph of $f(x) := \int_0^x g(t) dt$.
 - What are the absolute maximum and minimum values of $f(x)$?
 - Find a value of a such that $\int_a^x g(t) dt = f(x) - 2$.
- What is the area between the curves $f(x) = 6x^4 + 2$ and $g(x) = 5x^2 + 1$?
- What is the volume of the solid obtained by rotating the curve $y = x^2 - 4x + 5$ with $x \in [1, 3]$,
 - about the x -axis?
 - about the y -axis?
- What is the average value of $3x^2 + 1$ on $[1, 2]$?

