VERSION A

STUDENT NAME:

Solution to ~~test~~ quiz 12

Calculus 1550, section 20. Tuesday, October 7, 2003. Twelfth quiz

Let $A(t)$ be the number of Apes in a certain (fictional) forest, and $B(t)$ the number of a certain kind of Bug in the same forest. This particular Ape lives by eating only Bugs. It has been found that

$$\frac{dA}{dt} = A(0.05B - 0.000001)$$

and

$$\frac{dB}{dt} = B(0.005 - 0.0001A)^2.$$

Revised mark scheme:

1. [1 point] Write an inequality that means that the number of Apes is increasing.

[2]

$$\boxed{\frac{dA}{dt} > 0}$$

2. [1 point] Write an equation that means that the Bugs are extinct.

[2]

$$\boxed{B = 0}$$

3. [2 points] For what value of A is it possible for the Bug population to be stable (and nonzero)?

[2]

$B \neq 0$, $\frac{dB}{dt} = 0 \Rightarrow B(0.005 - 0.0001A) = 0$, & $B \neq 0$

$\Rightarrow A = \frac{0.005}{0.0001} = 50$ $\boxed{A = 50}$

4. [2 points] For what value of B is it possible for the Ape population to be stable (and nonzero)?

[2]

$A \neq 0$, $\frac{dA}{dt} = 0 \Rightarrow A(0.05B - 0.000001) = 0$, & $A \neq 0$

$\Rightarrow B = 0.000001/0.05 \approx 0$

Ape population stable when $B = 2 \times 10^{-5}$ but can't have fractional number of Bugs, so stability not possible.

5. [2 points] Now suppose

A increases by 1 (from your value in 3)

but

B decreases by 10 (from your value in 4).

What is the new value of $\frac{dB}{dt}$?

[1]

$A = 51$

$B = -10$

$$\frac{dB}{dt} = -10.(0.005 - 0.0051)^2 = -10.(-0.0001)^2 = -10^{-7} \approx 0$$

6. Which is more likely to become extinct, Apes or Bugs [1 point]? Why [1 point]?

[1]

Apes, since $\frac{dB}{dt}$ is always +ve, so B can't go extinct, unless this happens suddenly, but Apes go extinct if not enough bugs.

VERSION B

STUDENT NAME:

Solution to what should have been quiz 12

Calculus 1550, section 20. Tuesday, October 7, 2003. Twelfth quiz

Let $A(t)$ be the number of Apes in a certain (fictional) forest, and $B(t)$ the number of a certain kind of Bug in the same forest. This particular Ape lives by eating only Bugs. It has been found that

$$\frac{dA}{dt} = \cancel{A(0.05B - 0.000001)}$$

should have been

$$\frac{dA}{dt} = A(0.000001B - 0.05)$$

and

$$\frac{dB}{dt} = B(0.005 - 0.0001A)^2.$$

1. [1 point] Write an inequality that means that the number of Apes is increasing.

$$\frac{dA}{dt} > 0$$

2. [1 point] Write an equation that means that the Bugs are extinct.

$$B = 0$$

3. [2 points] For what value of A is it possible for the Bug population to be stable (and nonzero)?

$$A = 50$$

4. [2 points] For what value of B is it possible for the Ape population to be stable (and nonzero)?

$$\frac{dA}{dt} = 0 \Rightarrow A(0.000001\,B - 0.05) = 0 \quad \& \ A \neq 0$$
$$\Rightarrow B = 0.05/0.000001 = 50000$$

5. [2 points] Now suppose

A increases by 1 (from your value in 3) $\quad A = 51$

but

B decreases by 10 (from your value in 4). $\quad B = 49990$

What is the new value of $\frac{dB}{dt}$?

$$\frac{dB}{dt} = 49990 \times (0.005 - 0.0051)^2 \approx 0.0005$$

6. Which is more likely to become extinct, Apes or Bugs [1 point]? Why [1 point]?

Apes, since they need a huge number of Bugs to stay alive; bugs can never go extinct in this model, since $\frac{dB}{dt}$ is always +ve. If we start with too few Bugs, Apes go extinct.

STUDENT NAME:

Calculus 1550, section 20. Wednesday, October 8, 2003. Thirteenth quiz

Find the derivatives of the following functions. 2 points each.

1. $f(x) = \sin(x^2)$

$$f'(x) = (x^2)' (\sin)'(x^2)$$
$$= 2x \cos(x^2)$$

2. $f(x) = (\sin(x))^2$

$$f'(x) = (\sin)'(x)\; 2(\sin(x))$$
$$= \cos(x)\; 2\sin(x)$$
$$= 2\sin(x)\cos(x)$$
$$= \sin(2x)$$

3. $f(x) = \sin\left(\frac{1}{x}\right)$

$$f'(x) = \left(\frac{1}{x}\right)' (\sin)'\left(\frac{1}{x}\right)$$
$$= -1x^{-2} \cos\left(1/x\right)$$
$$= -\frac{\cos(1/x)}{x^2}$$

4. $f(x) = \sqrt{\sin(x)}$

$$f'(x) = \sin'(x)\; 1/2\,(\sin(x))^{-1/2}$$
$$= \cos(x)\; 1/2\,(\sin(x))^{-1/2}$$
$$= \frac{\cos(x)}{2\sqrt{\sin(x)}}$$

5. $f(x) = e^{x^4+x+1}$

$$f'(x) = (x^4+x+1)'\, e^{x^4+x+1}$$
$$= (4x^3+1)\, e^{x^4+x+1}$$