STUDENT NAME:

Calculus 1550, section 20. Tuesday, October 14, 2003. Fourteenth quiz

Use implicit differentiation to find $\frac{dy}{dx}$. 2 points each.

1. $x^{1/3} + y^{1/2} = 1$ (note, $y^{1/2} = 1 - x^{1/3}$)

$$\tfrac{1}{3}x^{-2/3} + \tfrac{1}{2}y^{-1/2}y' = 0$$

$$y' = -\tfrac{1}{3}x^{-2/3} \Big/ \tfrac{1}{2}y^{-1/2} = \boxed{-\tfrac{2}{3}\sqrt{y}/\sqrt[3]{x^2}} \left(= \frac{-\frac{2}{3}(1-\sqrt[3]{x^2})}{\sqrt[3]{x^2}}\right)$$

(can substitute $y^{1/2} = 1 - x^{1/3}$)

2. $y^2 + y = x^3 + 3x^2 + 5$

$$2yy' + y' = 3x^2 + 6x \Rightarrow y' = \boxed{\frac{3x^2+6x}{(2y+1)}}$$

3. $\cos(y) = \sin(x)$

$$-\sin(y)\,y' = \cos(x)$$

$$\boxed{y' = \frac{-\cos(x)}{\sin(y)}}$$

4. $xy = e^{x/y}$

$$xy' + y = y\frac{-xy'}{y^2}e^{x/y} = \frac{e^{x/y}}{y} - \frac{xy'}{y^2}e^{x/y}$$

$$\left(x + \frac{x}{y^2}e^{x/y}\right)y' = \frac{e^{x/y}}{y} - y$$

$$y' = \left(\frac{e^{x/y}}{y} - y\right)\Big/\left(x + \tfrac{x}{y^2}e^{x/y}\right) = \boxed{\frac{e^{x/y}y - y^3}{xy^2 + xe^{x/y}}}$$

5. $x\sin(y) = y\cos(x)$

$$\sin(y) + x\cos(y)y' = -y\sin(x) + \cos(x)y'$$

$$(x\cos(y) - \cos(x))y' = -y\sin(x) - \sin(y)$$

$$\boxed{y' = \frac{-y\sin(x) - \sin(y)}{x\cos(y) - \cos(x)}}$$