STUDENT NAME:

Calculus 1550, section 20. Wednesday, October 16, 2003. Sixteenth quiz

1. Differentiate the following functions (2 points each):

i. $f(x) = \ln(\sin(x))$

$$f'(x) = \frac{\sin'(x)}{\sin(x)} = \frac{\cos(x)}{\sin(x)} = \boxed{\cot(x)}$$

ii. $f(x) = \sin(\ln(x))$

$$f'(x) = \sin'(\ln(x))\ \ln'(x) = \cos(\ln(x)) \times \frac{1}{x} = \boxed{\frac{\cos(\ln(x))}{x}}$$

2. Use logarithmic differentiation to compute the derivatives of the following functions (3 points each):

i. $f(x) = (x+1)(x+2)^2(x+3)^3(x+4)^4$

Note ln turns × to + & ^ to ×

$\ln f(x) = \ln\left((x+1)(x+2)^2(x+3)^3(x+4)^4\right)$

$= \ln(x+1) + 2\ln(x+2) + 3\ln(x+3) + 4\ln(x+4)$ eg $\ln(a^b \times c^d) = b\ln(a) + d\ln(c)$

So $\frac{f'(x)}{f(x)} = \frac{1}{(x+1)} + \frac{2}{(x+2)} + \frac{3}{(x+3)} + \frac{4}{(x+4)}$

So $\boxed{f'(x) = (x+1)(x+2)^2(x+3)^3(x+4)^4\left(\frac{1}{(x+1)} + \frac{2}{(x+2)} + \frac{3}{(x+3)} + \frac{4}{(x+4)}\right)}$

ii. $f(x) = \sin(x)^{\sin(x)}$

use product rule

$\ln f(x) = \ln\left(\sin(x)^{\sin(x)}\right) = \sin(x)\ln(\sin(x))$

$\frac{f'(x)}{f(x)} = \sin(x)\left(\ln(\sin(x))\right)' + \sin'(x)\ln(\sin(x))$

this ↑ deriv. is calculated in 1.i.

$= \sin(x)\frac{\cos(x)}{\sin(x)} + \cos(x)\ln(\sin(x)) = \cos(x)\left(1 + \ln(\sin(x))\right)$

So $\boxed{f'(x) = \sin(x)^{\sin(x)}\cos(x)\left(1 + \ln(\sin(x))\right)}$