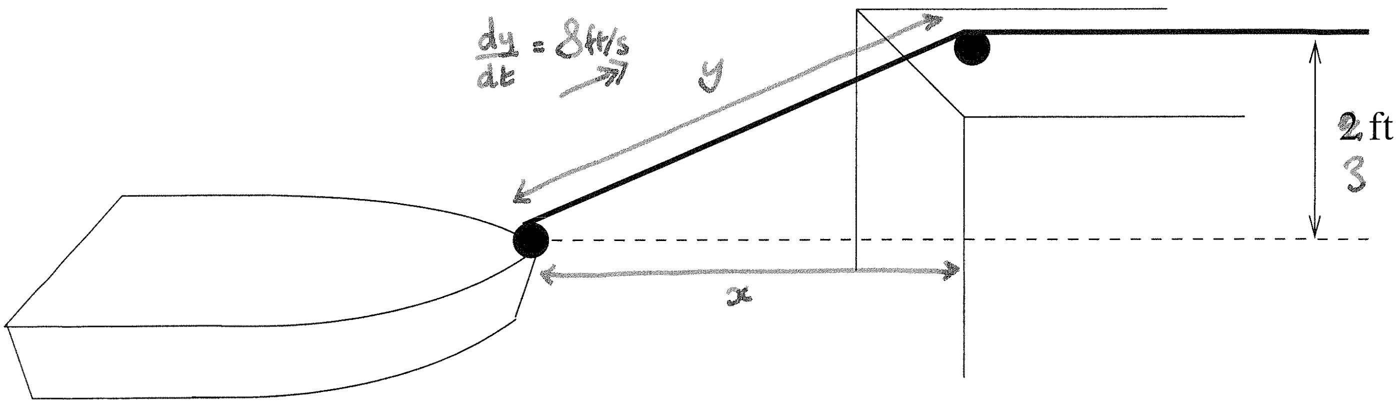
STUDENT NAME:

Calculus 1550, section 20. Tuesday, October 21, 2003. Seventeenth quiz

A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 3 ft higher than the bow of the boat. If the boat is pulled in at a rate of 8 ft/s, how fast is the boat approaching the dock when it is 4ft from the dock?



Points are given for 1) making your notation clear, labling the above diagram correspondingly, and stating values the variables and their derivatives take [3 points]; 2) making it clear what relation you find which needs to be differentiated [2.5 points]; 3) differentiating correctly [2.5 points]; 4) obtaining the final answer [2 points].

let x = distance of boat from dock, horizontally : want to find $\frac{dx}{dt}$ when $x = 4$ ft.
y = distance between the 2 pulleys: $\frac{dy}{dt} = 8$ ft/s.

Relation

$$x^2 + 3^2 = y^2 \qquad \text{so, at } x = 4, \; y = 5$$

differentiate:

$$2x\frac{dx}{dt} + 0 = 2y\frac{dy}{dt}$$

$$\Rightarrow \frac{dx}{dt} = \frac{y}{x}\frac{dy}{dt}$$

at $x = 4$, $y = 5$ & $dy/dt = 8$ so $\frac{dx}{dt} = \frac{5}{4} \times 8 = 10$ ft/s

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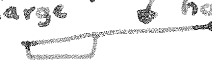
when x large: rope almost horizontal



Remark: when x is very large, "3" is negligible, so $x \approx y$ & $dx/dt \approx dy/dt = 8$, i.e. $\lim_{x \to \infty} \frac{dx}{dt} = 8$ ft/s.

when x is very small, we almost have a "pendulum":
y changes very little $\Rightarrow$ x changes a lot (relative to x)
so $\lim_{x \to 0} \frac{dx}{dt} = \infty$.

Trend: the closer to the dock, the faster the boat. Always faster than 8 ft/s

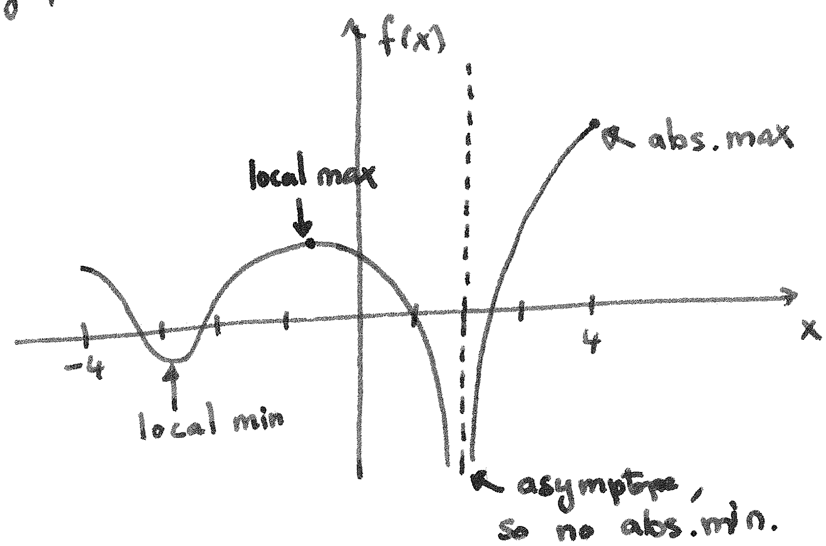
STUDENT NAME:

Calculus 1550, section 20. Wednesday, October 22, 2003. Eighteenth quiz

1. [4 points] Sketch the graph of a function on the interval $[-4, 4]$ which has on this interval:

   1 local minimum
   no absolute minimum
   1 local maximum
   an absolute maximum, which is not at a critical value

Many possible solutions, e.g.



2. [6 points] Find the absolute maximum and absolute minimum values of the function

$$f(x) = \frac{x}{x^2+1}$$

on the interval $[0, 2]$.

$$f'(x) = \frac{(x^2+1) - 2x \times x}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$ } 2 points for correct derivation

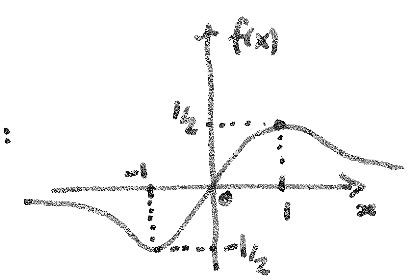
so critical values at 1 & -1, when $f'(x)=0$.
note, x^2+1 always +ve, so $f'(x)$ always defined

only $1 \in [0,2]$.

$f(0) = 0$ ← smallest
$f(1) = 1/2 = 0.5$ ← largest
$f(2) = 2/5 = 0.4$ ◁

} 2 points for evaluating at correct critical val & end pts.

sketch:



absolute max on $[0,2]$ is 0.5, at $x=1$
" min " $[0,2]$ is 0 at $x=0$.

1 point for abs max
1 point for abs min