

# STUDENT NAME:

Calculus 1550, section 20. Wednesday, October 29, 2003. Twenty-first quiz.

Let  $f(x)$  be the function

$$f(x) = x^6 - 3x^4 + 3x^2.$$

1. [1/2 point] What is the domain of  $f$ ?  $\mathbb{R} = (-\infty, \infty)$

2.a. [1/2 point] What is  $f(0)$ ? 0

2.b. [1/2 point] For what  $x$  does  $f(x) = 0$ ? 0

$$f(x) = 0 \Rightarrow x^6 - 3x^4 + 3x^2 = 0$$

$$\Rightarrow x^2(x^4 - 3x^2 + 3) = 0$$

$$\Rightarrow x = 0 \text{ or } x^4 - 3x^2 + 3 = 0$$

$$\Rightarrow x^2 = \frac{3 \pm \sqrt{9-12}}{2} \leftarrow \text{not real, so } 0 \text{ only place where } f(x) = 0$$

3. [1/2 point] Is  $f$  odd, even, or periodic, or have no symmetries? \_\_\_\_\_

even, since all exponents (6, 4 & 2) are even

4. [1/2 point] Does  $f$  have horizontal or vertical asymptotes, and if so, what are they? none

$$\begin{aligned} f'(x) &= 6x^5 - 12x^3 + 6x = 6(x^4 - 2x^2 + 1)x \\ &= 6x(x^2 - 1)(x^2 - 1) \\ &= 6x(x-1)(x+1)^2 \end{aligned}$$

5.a. [1/2 point] What are the critical numbers of  $f(x)$ ? 0, -1, 1

$$f'(x) = 0 \text{ at } x = 0 \text{ or } x = \pm 1$$

5.b. [1/2 point] On which intervals is  $f'(x)$  increasing? (0, \infty)

$$f'(x) = 6x(x^2 - 1)^2$$

$$6(x^2 - 1)^2 \geq 0, \text{ so } f'(x) \geq 0 \text{ depending on } x \geq 0$$

5.c. [1/2 point] What is the value of  $f(x)$  at local maxima and minima?  $f(0) = 0$  local min.

Sign of  $f'(x)$ :

$(-\infty, -1)$   $(-1, 0)$   $(0, 1)$   $(1, \infty)$

$f'(x)$

$-$   
no sign change  
 $\Rightarrow$  inflection point

$-$   $\uparrow$   $+$   
only sign change,  
from  $-ve$  to  $+$ ,  
so minimum

$+$   $\uparrow$   $+$   
no sign change  
 $\Rightarrow$  inflection pt.