6.a. [1/2 point] For what x is $f''(x) = 0$? $1, -1, -1/\sqrt{5}, 1/\sqrt{5}$

$f'(x) = 6(x^5 - 2x^3 + x)$

so $f''(x) = 6(5x^4 - 6x^2 + 1) = 6(5x^2 - 1)(x^2 - 1)$

so $f''(x) = 0$ at $x = \pm 1$ & $\pm 1/\sqrt{5}$

6.b. [1/2 point] On what intervals is $f'(x)$ concave up? $(-\infty, -1) \cup (-1/\sqrt{5}, 1/\sqrt{5}) \cup (1, \infty)$

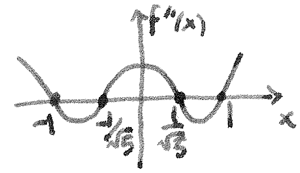
(Typo - should be f!)

between the zeros, $f''(-2) = f''(2) = 342$

$f''(-0.3) = f''(0.3) \approx -9.5$

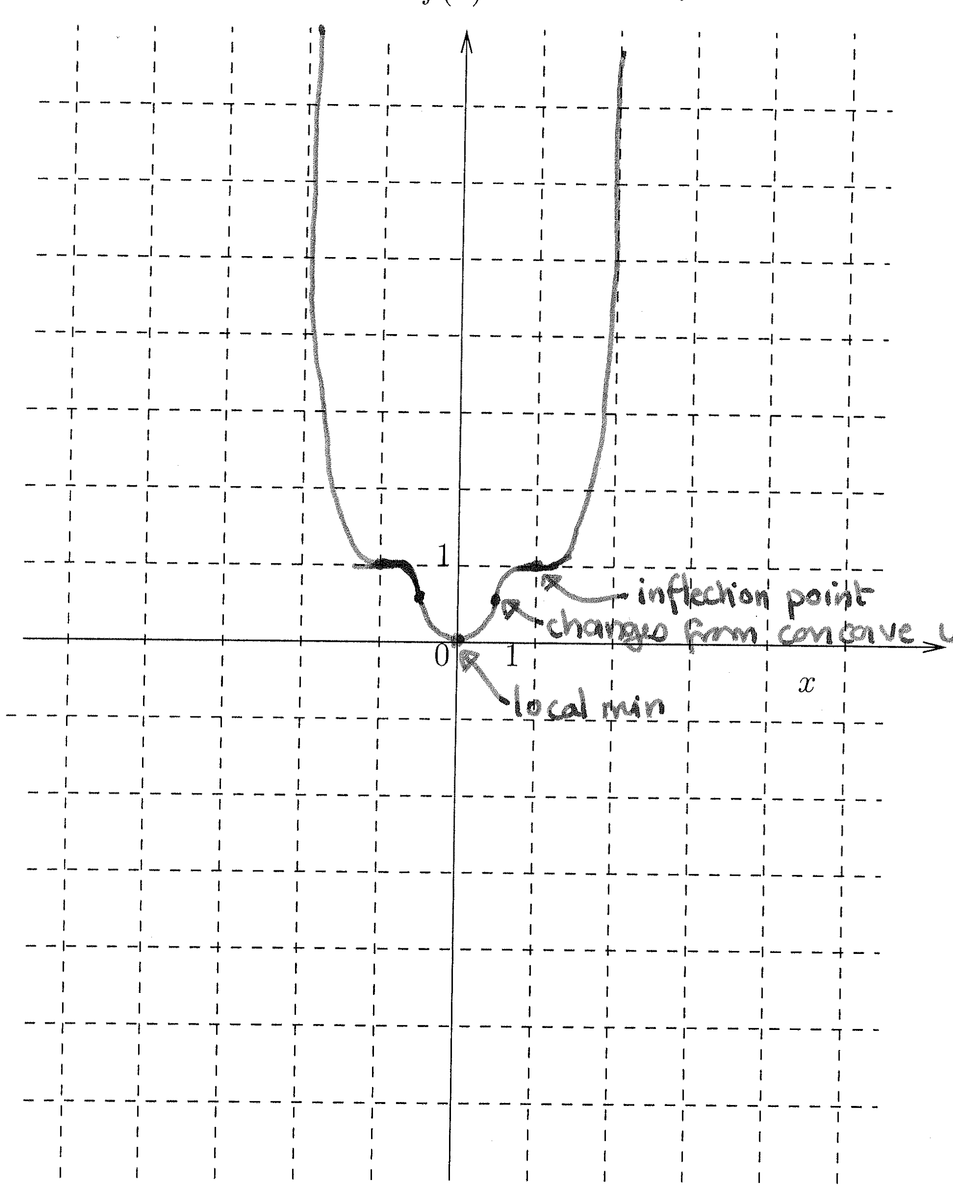
$f''(0) = 6$

$f''(x)$ looks like:



7. [5 points] Sketch a graph of $f(x)$, showing the above features.

$f(x) = x^6 - 3x^4 + 3x^2$



some values:

$f(1) = 1$

$f(2) = 28$

$f(1/2) \approx 0.58$

$f(1/\sqrt{5}) = 0.488$

$1/\sqrt{5} \approx 0.447$

Remark: f' is increasing where f'' is +ve, ie $(-\infty, -1) \cup (-1/\sqrt{5}, 1/\sqrt{5}) \cup (1, \infty)$

f' is concave up when f''' +ve. $f'''(x) = 6(20x^3 - 12x) = 24x(5x^2 - 3)$

so $f''' = 0$ at $x = 0$ & $\pm\sqrt{3/5}$

& f''' +ve on $(-\sqrt{3/5}, 0) \cup (\sqrt{3/5}, \infty)$