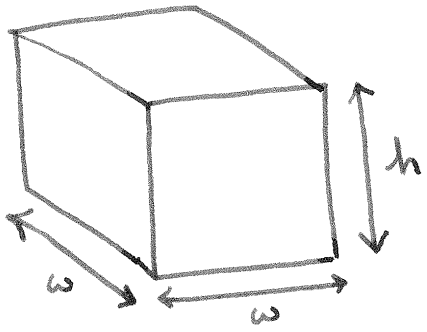
STUDENT NAME:

Calculus 1550, section 20. Thursday, November 6, 2003. Twenty-second quiz.

A box with a square base and open top has to have volume 0.5 m^3.
Material for the base is \$10 per square meter,
material for the sides is \$6 per square meter.
What is the height of the cheapest box?

Show your working, including making your notation clear, and labeling a diagram.



Let h = height of box
w = length & width of base.

so V = volume = $1/2 = hw^2$

C = cost = cost of sides + cost of base

= 4 × cost of each side + cost of base

= 4 × 6 × area of a side + 10 × area of base

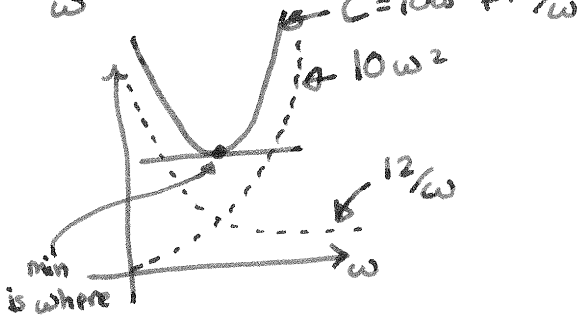
$= 4 \times 6 \times wh + 10w^2$

$C = 24wh + 10w^2$

Since $1/2 = hw^2$, $h = \frac{1}{2w^2}$

so $C = \frac{24w}{2w^2} + 10w^2 = \frac{12}{w} + 10w^2$

want to minimize C. graph of C looks like:



$\frac{dC}{dw} = \frac{-12}{w^2} + 20w$

so when $\frac{dC}{dw} = 0$, $\frac{12}{w^2} = 20w$

$\Rightarrow \frac{12}{20} = w^3 \Rightarrow w = \sqrt[3]{\frac{12}{20}} \approx 0.8434$

so width of cheapest box base is 0.8434

height is $h = \frac{1}{2w^2} = \frac{1}{2 \times 0.8434^2} \approx 0.703$