

STUDENT NAME:

Calculus 1550, section 20. Thursday, November 13, 2003. Twenty-fourth quiz.

Let

$$f(x) = 4x^3 - 27x^2 + 50x,$$

sketched right.

1. Sketch three rectangular strips which approximate the area under the curve between $x = 1$ and $x = 4$. Compute the area of each strip, and use this to estimate the area under the curve between $x = 1$ and $x = 4$.

Rectangles shown have areas

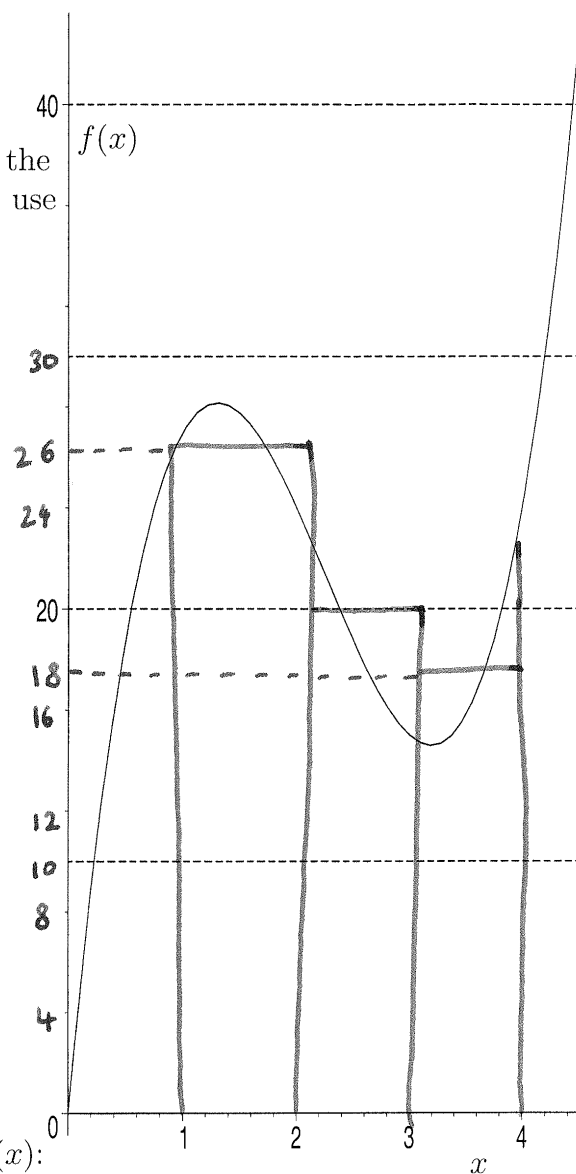
$$26 \times 1 = 26$$

$$20 \times 1 = 20$$

$$18 \times 1 = 18$$

so total area of rectangles is 64

(Using different rectangles will give different approximations. If your rectangles all have width 1, the maximum area would be 76.16
minimum " " " 53.59
so anything in this range would be an acceptable estimate)



2. Compute the following integral by finding the anti-derivative of $f(x)$:

$$\int_1^4 (4x^3 - 27x^2 + 50x) dx$$

$$= [x^4 - 9x^3 + 25x^2]_1^4$$

$$= 4^4 - 9 \cdot 4^3 + 25 \cdot 4^2 - 1 + 9 - 25$$

$$= 4^2(4^2 - 9 \cdot 4 + 25) - 17$$

$$= 16 \times (4 \times 4 - 4 \times 9 + 25) - 17$$

$$= 16(-4 \times 5 + 25) - 17 = 16 \times 5 - 17 = 80 - 17 = 63$$

(This is for students who don't have a calculator,
to show it can be done!)

STUDENT NAME: *Additional working to show max & min areas.*
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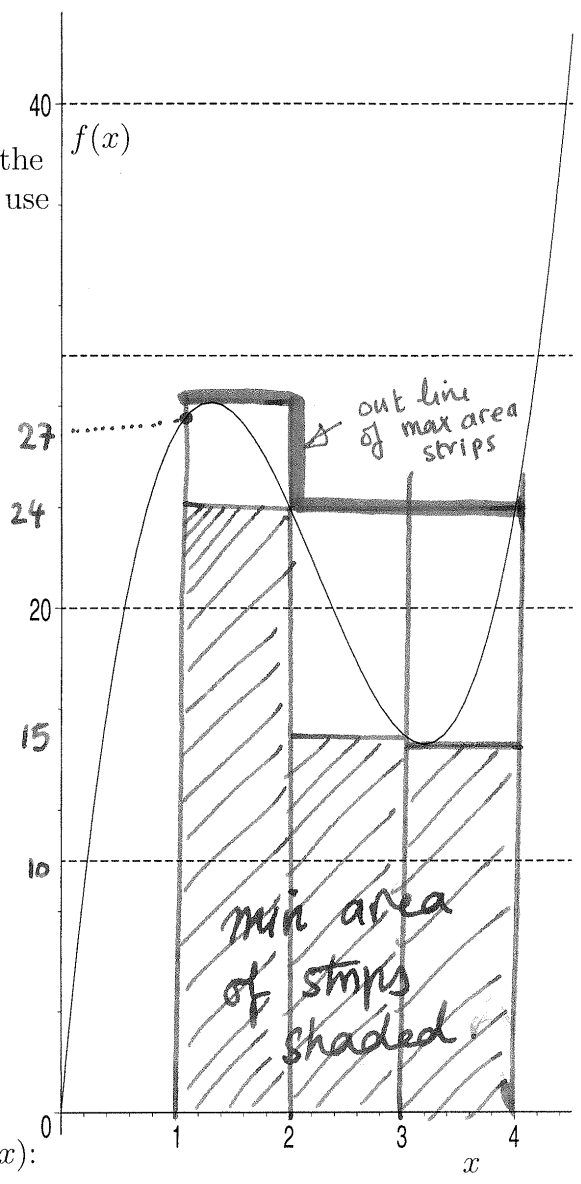
Let $f(x) = 4x^3 - 27x^2 + 50x$,

sketched right.

1. Sketch three rectangular strips which approximate the area under the curve between $x = 1$ and $x = 4$. Compute the area of each strip, and use this to estimate the area under the curve between $x = 1$ and $x = 4$.

$f(1) = 27$
 $f(2) = 24$
 $f(3) = 15$
 $f(4) = 24$
 $f'(x) = 12x^2 - 54x + 50$
 min & max values of $f(x)$ are at $x = \frac{54 \pm \sqrt{54^2 - 4 \times 50 \times 12}}{2 \times 12}$
 ≈ 1.3035 & 3.1964

so max area is $28.16 + 24 + 24 = 76.16$
 min area is $24 + 15 + 14.59 = 53.59$
 your estimate should probably be between these.



2. Compute the following integral by finding the anti-derivative of $f(x)$:

$$\int_1^4 (4x^3 - 27x^2 + 50x) dx$$