

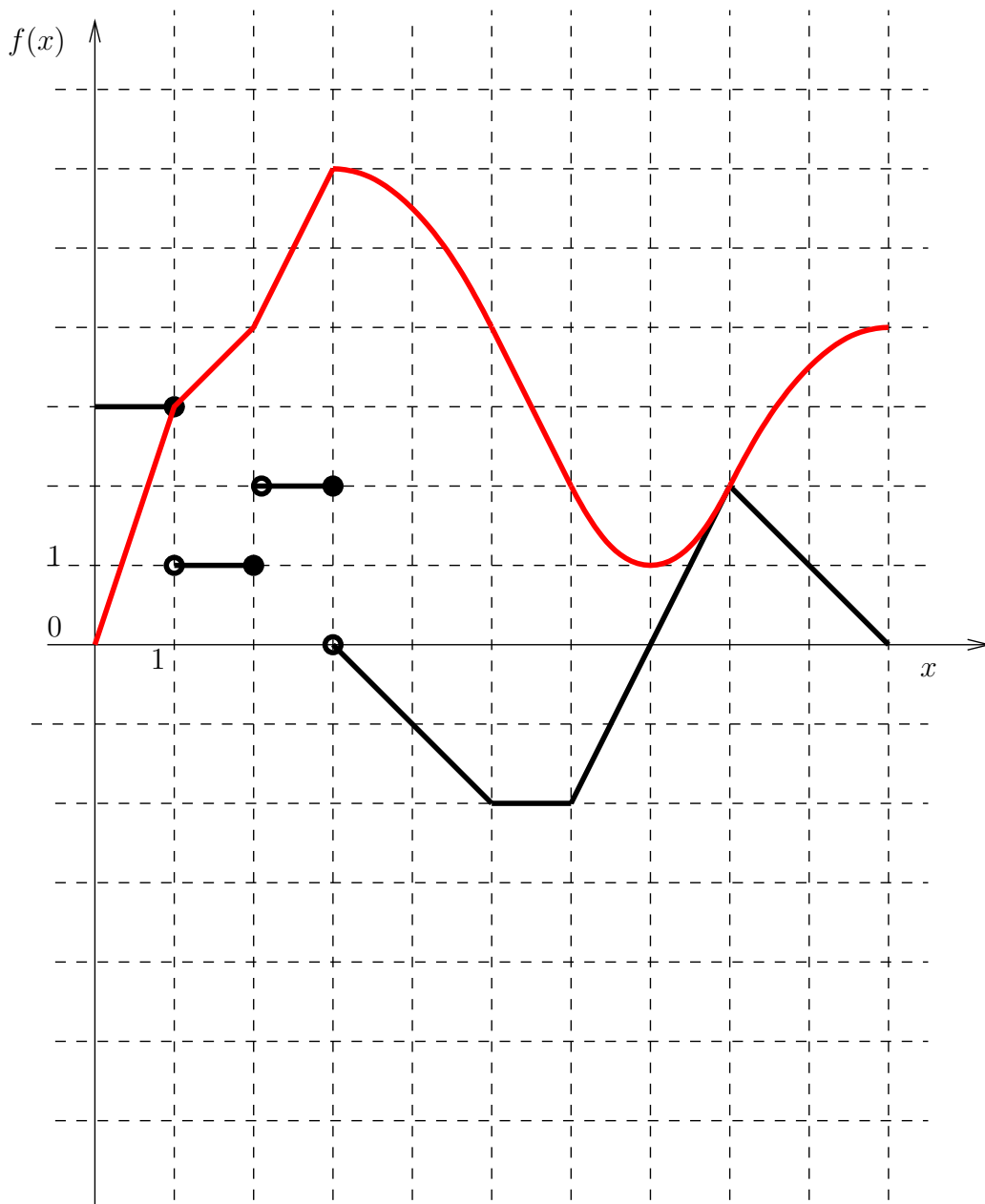
STUDENT NAME:

Calculus 1550, section 20. Tuesday, November 18, 2003. Twenty-fifth quiz.

Below the graph of a function $f(x)$ is sketched.

1. [6 points] On the same grid, Sketch a graph of the function

$$g(x) = \int_0^x f(t) dt$$



1(Answer). On the left, $f(x)$ is in black, and $g(x)$ in red.

Explanation of how $g(x)$ is computed:

$g(x)$ is the area under the curve from 0 to x .

So, for example, $g(1) = 3$, (area of three squares up to $x = 1$) and $g(2) = 4$ (the first three squares, plus one more, from 1 to 2). You can compute all the areas at $g(1), (2), \dots, g(10)$ just by computing areas of triangles and rectangles. In between, if $f(x)$ is constant, $g(x)$ is linear, and when $f(x)$ is linear, $g(x)$ is quadratic.

2. [1 point] What is the maximum value of $g(x)$ on $[0, 10]$?

2(Answer). the maximum, from the sketch, is at $x = 3$. (Note that at this value of x , we have $f(3) = 0$.)
Since $g(3) = 6$, the maximum value is 6.

3. [1 point] What is the minimum value of $g(x)$ on $[0, 10]$?

3(Answer). the minimum, from the sketch, is at $x = 0$. (Note, there is also a local minimum at 7, where $g(x) = 1$.)
Since $g(0) = 0$, the minimum value is 0.

4. [2 points] Suppose

$$\int_a^x f(t) dt = g(x) - 5$$

What is a ?

4(Answer). This means that $\int_a^x f(t) dt = \int_0^x f(t) dt - 5$

so $5 = \int_0^x f(t) dt - \int_a^x f(t) dt = \int_0^a f(t) dt = g(a)$

(Because the area from 0 to x minus the area from a to x is the area from 0 to a)

So, where does $g(a) = 5$? from the sketch, this is at $a = 2.5$.

($a = 4.5$ another possible solution).