STUDENT NAME:

Calculus 1550, section 20. Wednesday, November 25, 2003. Twenty-sixth quiz

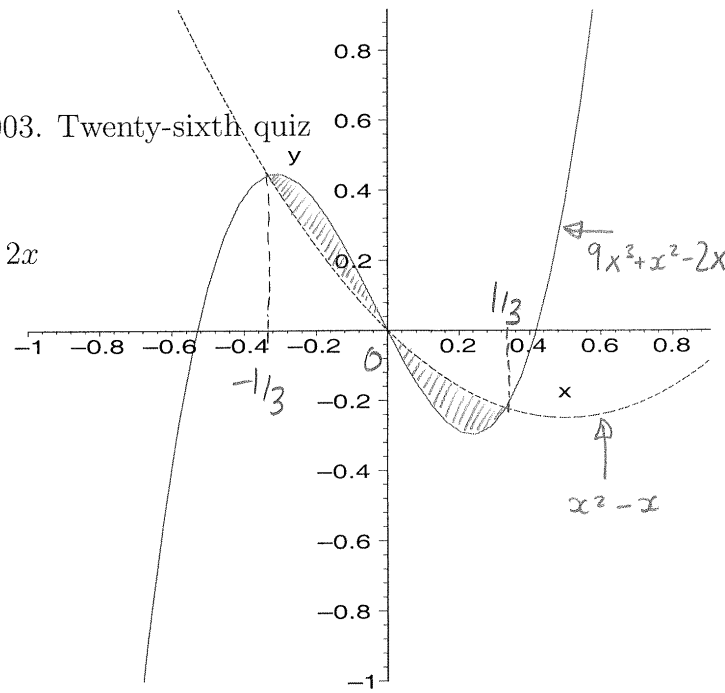
1. What is the area between the curves

$$y = f(x) := 9x^3 + x^2 - 2x$$

and

$$y = g(x) := x^2 - x,$$

which is sketched right?



$f(x) = g(x)$

at $9x^3 + x^2 - 2x = x^2 - x$

$\Rightarrow 9x^3 = x \Rightarrow x = 0$ or $9x^2 = 1$

$\Rightarrow x^2 = 1/9$

$\Rightarrow x = \pm 1/3$

so, area is $\int_{-1/3}^{1/3} |f(x) - g(x)|\,dx = \int_{-1/3}^{1/3} |9x^3 - x|\,dx$

$$= \left|\int_{-1/3}^{0} (9x^3 - x)\,dx\right| + \left|\int_{0}^{1/3} (9x^3 - x)\,dx\right|$$

$$= \left|\left[9/4\,x^4 - 1/2\,x^2\right]_{-1/3}^{0}\right| + \left|\left[9/4\,x^4 - 1/2\,x^2\right]_{0}^{1/3}\right|$$

$$= \left|9/4\ 1/81 - 1/2\ 1/9\right| + \left|9/4\ 1/81 - 1/2\ 1/9\right| = \frac{1}{4}\cdot\frac{1}{9} + \frac{1}{4}\cdot\frac{1}{9} = \frac{1}{2}\cdot\frac{1}{9} = \frac{1}{18} \approx 0.0555$$

$$= \left|\frac{1}{4}\cdot 9 - \frac{1}{2}\cdot\frac{1}{9}\right| + \left|\frac{1}{4}\cdot 9 - 1/2\cdot 1/9\right|$$

note, we could also use the fact that $9x^3 - x$ is odd, so $|9x^3 - x|$ is even,

so $\int_{-1/3}^{1/3} |f(x) - g(x)|\,dx = 2\left|\int_{0}^{1/3} (9x^3 - x)\,dx\right| = 2\int_{0}^{1/3} (x - 9x^3)\,dx = 2\left[1/2\,x^2 - 9\frac{x^4}{4}\right]_{0}^{1/3} = \frac{1}{18}$

(in picture, can see that $x^2 - x \geq 9x^3 + x^2 - 2x$ on $[0, 1/3]$ ie $x - 9x^3 \geq 0$ on $[0, 1/3]$)