

STUDENT NAME:

Calculus 1550, section 20. Tuesday December 2, 2003. Twenty-seventh quiz

Let

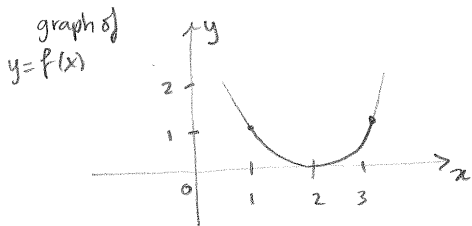
$$f(x) = (x-2)^2.$$

Let A be the region defined by

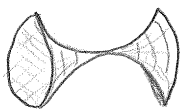
$$A := \{(x, y) \in \mathbf{R}^2 \mid x \in [1, 3], 0 \leq y \leq f(x)\}.$$

1. Sketch the solid obtained by rotating A about the x -axis. What is its volume?

$$f(1) = -1^2 = 1, \quad f(3) = 1^2 = 1, \quad f(2) = 0$$



shape obtained by rotating about x -axis:



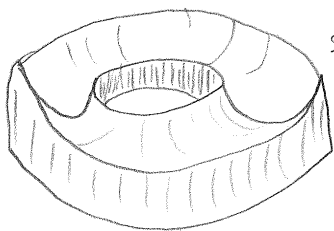
$$\text{Volume} = \int_1^3 \pi (x-2)^4 dx$$

$$\text{let } u = x-2 \\ du = dx$$

$$\text{if } x=1, u=-1, \text{ if } x=3, u=1$$

$$= \int_{-1}^1 \pi u^4 du = \left[\pi \frac{1}{5} u^5 \right]_{-1}^1 = \frac{\pi}{5} (1^5 - (-1)^5) = \frac{2\pi}{5}$$

2. Sketch the solid obtained by rotating A about the y -axis. What is its volume?



shape obtained by rotating about y -axis

$$\text{Volume} = \int_1^3 2\pi x (x-2)^2 dx = \int_1^3 2\pi (x^3 - 4x^2 + 4x) dx$$

$$= 2\pi \left[\frac{1}{4} x^4 - \frac{4}{3} x^3 + 2x^2 \right]_1^3$$

$$= 2\pi \left(\frac{1}{4} 81 - \frac{4}{3} \cdot 27 + 2 \cdot 9 - \left(\frac{1}{4} - \frac{4}{3} + 2 \right) \right)$$

$$= 2\pi \left(\frac{1}{4} \cdot 80 - 36 + 18 + \frac{4}{3} - 2 \right) = 2\pi \left(20 - 9\frac{2}{3} \right) = 2\pi \left(0\frac{1}{3} \right) = \frac{62\pi}{3}$$

STUDENT NAME:

Calculus 1550, section 20. Wednesday December 3, 2003. Twenty-eighth quiz

- i. What is the average value f_{ave} of the function $f(x) = e^x$ on the interval $[1, 3]$?
- ii. Find some $c \in [1, 3]$ with $f(c) = f_{\text{ave}}$.

$$i) \quad f_{\text{ave}} = \frac{1}{2} \int_1^3 e^x dx = \frac{1}{2} [e^x]_1^3 = \frac{1}{2} (e^3 - e) \approx 8.684$$

ii)

$$\text{if } f(c) = f_{\text{ave}},$$

$$e^c = \frac{1}{2} (e^3 - e) \approx 8.684$$

$$\text{so } c = \ln\left(\frac{1}{2} (e^3 - e)\right) \approx \ln(8.684) \approx 2.161$$

(note we can also write

$$c = \ln\left(\frac{1}{2} e (e^2 - 1)\right)$$

$$= \ln\left(\frac{1}{2} e (e-1)(e+1)\right)$$

$$= \ln(e) + \ln(e-1) + \ln(e+1) - \ln(2)$$

$$\approx 2.161$$