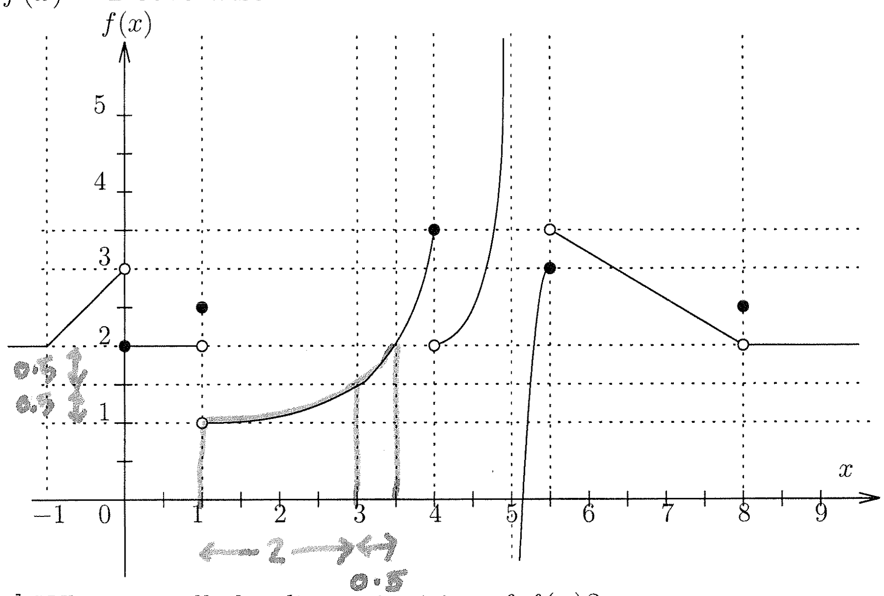
STUDENT NAME:

Calculus 1550, section 20. Thursday 11 September, 2003. First test.

This test paper has 5 pages. Points for parts of questions are given in square brackets. You have 50 minutes.

Q1. A function $f(x)$ has domain $\mathbb{R}$, and is given by the following graph for $x \in [-1, 9]$, and by $f(x) = 2$ otherwise.



i. [5] What are all the discontinuities of $f(x)$?

0, 1, 4, 5, 5.5, 8

ii. [15] Complete the following table:

a	$f(a)$	$\lim_{x\to a^-} f(a)$	$\lim_{x\to a^+} f(a)$	$\lim_{x\to a} f(a)$	right continuous at a?	left continuous at a?
-1	2	2	2	2	YES	YES
4	3.5	3.5	2	DNE	NO	YES
8	2.5	2	2	2	NO	NO

iii. [5] Find a number δ so that $|f(x) - 1.5| < 0.5$ whenever $|x - 3| < \delta$.

if $1 < x < 3.5$, then $|f(x) - 1.5| < 0.5$

$3 - 1 = 2$, $3.5 - 3 = 0.5$, $\min(0.5, 2) = 0.5$

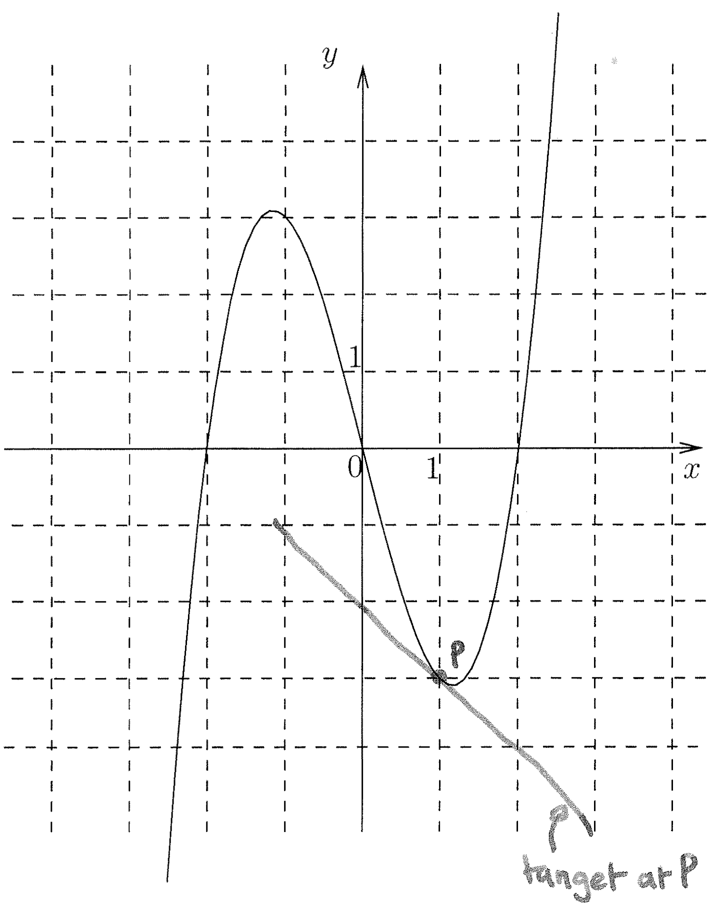
so take $\delta = 0.5$

Q2. Let C be a curve given by $y = x(x+2)(x-2)$. This is sketched below.
Let P be a point on C with x-coordinate 1.
Let m be the slope of the tangent to C at P.
If Q is another point on C, then m_{PQ} is the slope of the secant through P and Q.

1. [3] Plot P on the graph, and draw the tangent to C at P.

1. [1] What is the y coordinate of P? -3
2. [9] Compute the slope of the secants PQ, with Q as in the table below.
3. [4] What are the bounds on m you have computed? (Write inequalities.) $-1.03 \leq m \leq -0.7$
4. [2] What do you guess the value of m is? -1
5. [6] Write an equation for the tangent of C at P, using your estimate for the slope.

$\frac{y - -3}{x - 1} = -1$ so $y + 3 = -x + 1$ $\Rightarrow$ $\boxed{y = -x - 2}$



Fill in values to at least 3 DECIMAL PLACES

Q		m_{PQ}
x coord	y coord	
2	$2 \times 4 \times 0 = 0$	$\frac{\Delta y}{\Delta x} = \frac{3}{1} = 3$
1.001	$1.001 \times 3.001 \times -1.001 \doteq -3.007$	$\frac{-0.007}{0.001} = -0.7$
0.99	$0.99 \times 2.99 \times -1.01 \doteq \cancel{-2.990}$	$\frac{+0.0103}{-0.01} = -1.03$

$\doteq 2.9897$

Q3. Evaluate the following limits, using limit laws and techniques for computing limits exactly. Show your working, or explain your reasoning, but it is not necessary to write down which laws you are using. If a limit does not exist, explain why not.

i. [7] $\lim_{x\to 2} \sqrt{x^2+x+3}$

$$= \sqrt{\lim_{x\to 2}(x^2+x+3)} = \sqrt{4+2+3} = \sqrt{9} = 3$$

ii. [7] $\lim_{x\to 2} \dfrac{x^3-8}{x^2-4}$

$$= \lim_{x\to 2} \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x+2)}$$

$$= \lim_{x\to 2} \frac{(x^2+2x+4)}{(x+2)} = \frac{2^2+4+4}{2+2} = 3$$

iii. [7] $\lim_{x\to 0} \dfrac{(3+x)^2-9}{x}$

$$= \lim_{x\to 0} \frac{(\cancel{9}-6x+x^2)-\cancel{9}}{x}$$

$$= \lim_{x\to 0} \frac{-6x+x^2}{x}$$

$$= \lim_{x\to 0} (-6+x) = -6$$

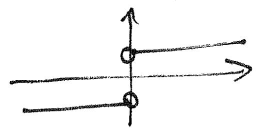
iv. [7] $\lim_{x \to 0} \frac{|x|}{x}$

does not exist, since if $x > 0$, $\frac{|x|}{x} = 1$

if $x < 0$, $\frac{|x|}{x} = -1$

so $\lim_{x \to 0^+} \frac{|x|}{x} = +1$, $\lim_{x \to 0^-} \frac{|x|}{x} = -1$, these are <u>not</u> equal.

graph:



v. [7] $\lim_{x \to 1} \frac{\sqrt{5-x} - 2}{x - 1}$

$$= \lim_{x \to 1} \frac{(\sqrt{5-x} - 2)(\sqrt{5-x} + 2)}{(x-1)(\sqrt{5-x} + 2)}$$

$$= \lim_{x \to 1} \frac{(5 - x - 4)}{(x-1)(\sqrt{5-x} + 2)}$$

$$= \lim_{x \to 1} \frac{(1-x)}{(x-1)(\sqrt{5-x} + 2)}$$

$$= \lim_{x \to 1} \frac{-1}{(\sqrt{5-x} + 2)} = \frac{-1}{\sqrt{4} + 2} = \frac{-1}{2+2} = -1/4$$

Q4. Given some numbers a and b, define a function

$$f_{ab}(x) = \frac{x^3 + ax + b}{x^2 + 4x}.$$

Fill in the blanks in the table, to give values of a and b to make the last column correct. Write one number in each space. [3 points for each entry].

a	b	$\lim_{x\to 0} f_{ab}(x)$
5	1	DNE ~~∞~~
8	0	2
0	0	0

if $b \neq 0$, $x^3 + ax + b \neq 0$ at $x = 0$

but $x^2 + 4x = 0$ at $x = 0$,

So, if $b \neq 0$, for any value of a, $\lim_{x\to 0} f_{ab}(x)$ does not exist.

So any value of a OK in 1st col, top row.

Remaining b must both be 0.

if $b = 0$, $\lim_{x\to 0} \frac{x^3 + ax}{x^2 + 4x} = \lim_{x\to 0} \frac{x^2 + a}{x + 4} = \frac{a}{4}$

So for $\frac{a}{4} = 2$, $a = 8$

for $\frac{a}{4} = 0$, $a = 0$