STUDENT NAME:

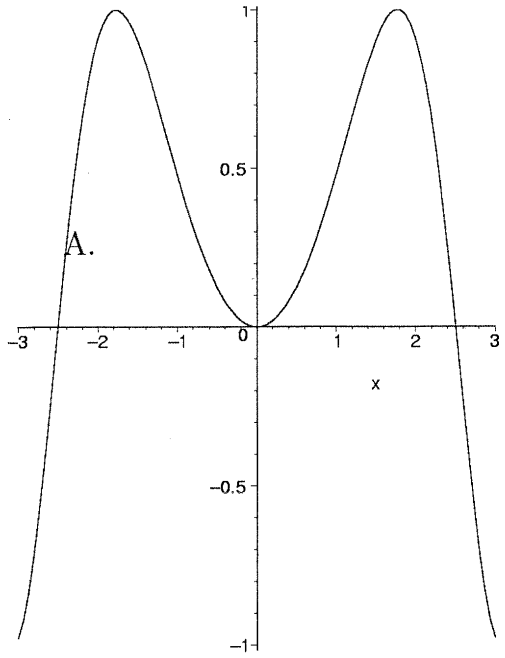
Calculus 1550, section 20. Second test. Thursday, October 9, 2003.

This test paper has ~~8~~ 6 pages. Points per question are given in square brackets.

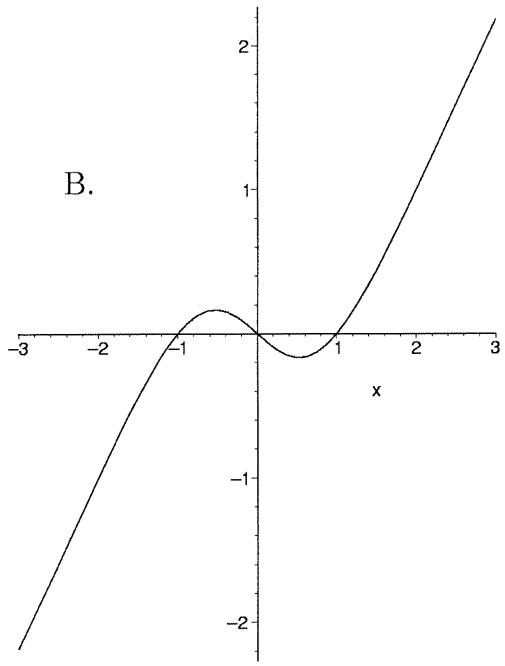
1. [24 points] On page 2, the graphs of 6 functions are shown.
On page 3, the graphs of their derivatives are drawn, in some order.
Match up the correct derivatives with each function, and write your answers in the table:

A	5
B	1
C	4
D	2
E	6
F	3

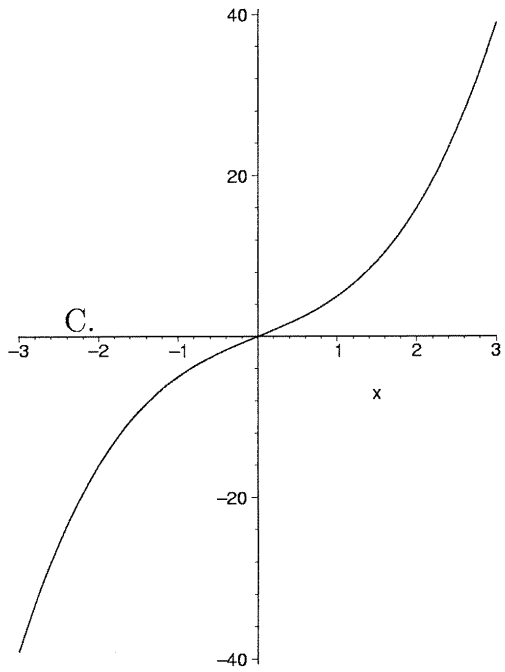
A.


1
0.5
−0.5
−1
−3
−2
−1
0
1
2
3
x

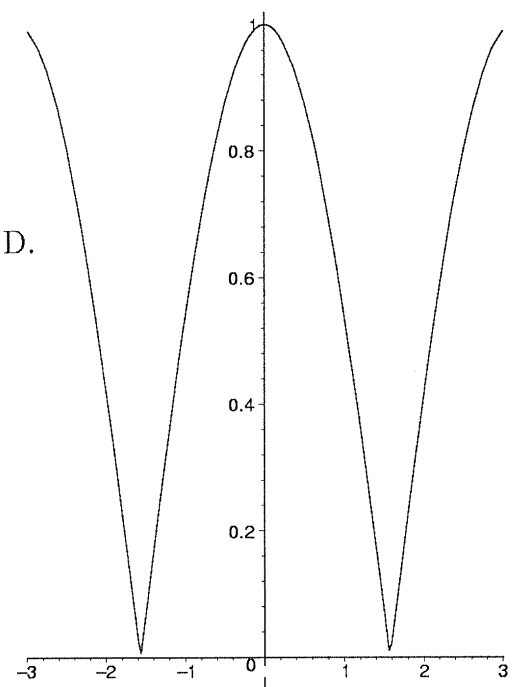
B.


2
1
−1
−2
−3
−2
−1
1
2
3
x

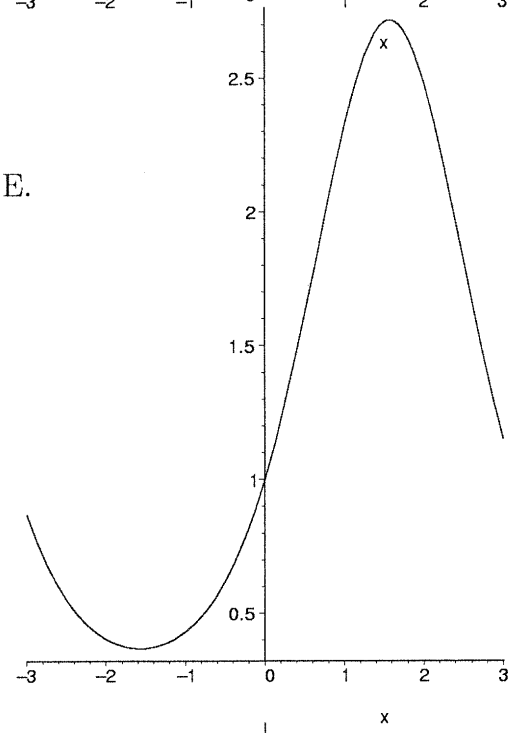
C.


40
20
−20
−40
−3
−2
−1
1
2
3
x

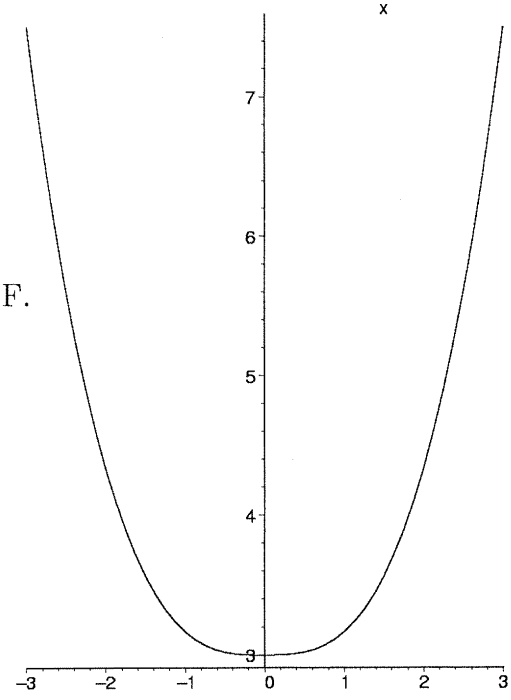
D.


1
0.8
0.6
0.4
0.2
−3
−2
−1
0
1
2
3

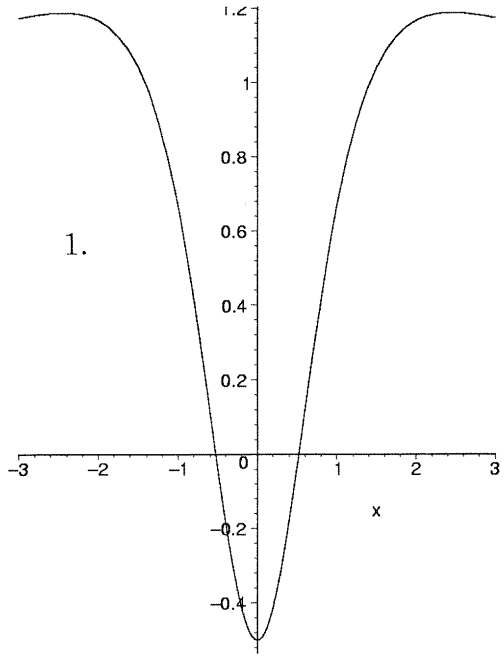
E.


x
2.5
2
1.5
1
0.5
−3
−2
−1
0
1
2
3
x

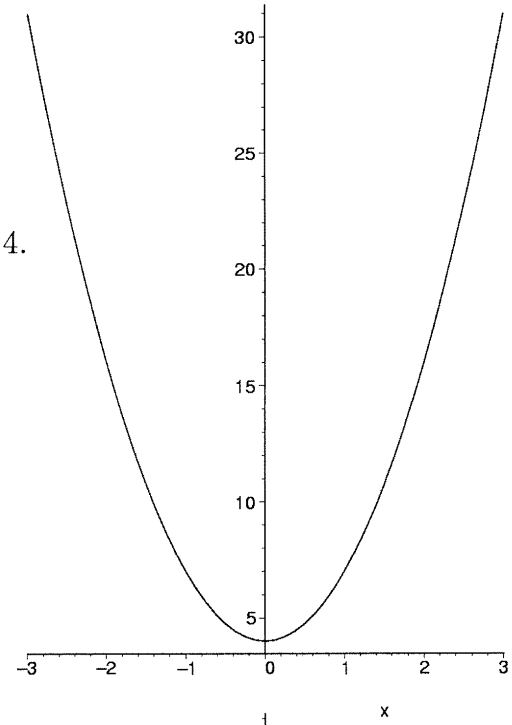
F.


7
6
5
4
3
−3
−2
−1
0
1
2
3

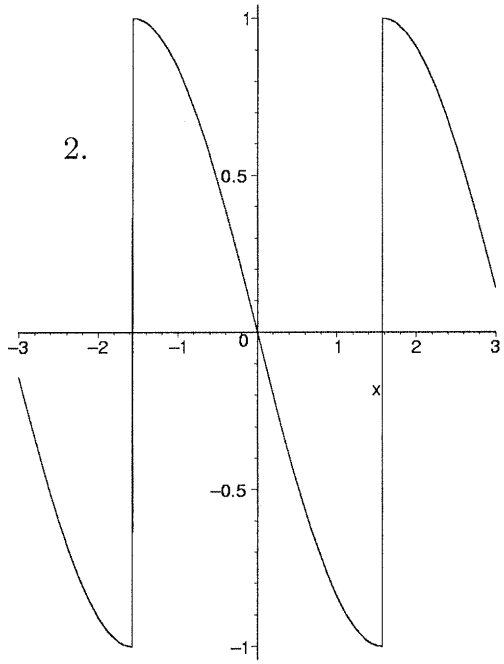
1.


1
0.8
0.6
0.4
0.2
−0.2
−0.4
−3
−2
−1
0
1
2
3
x

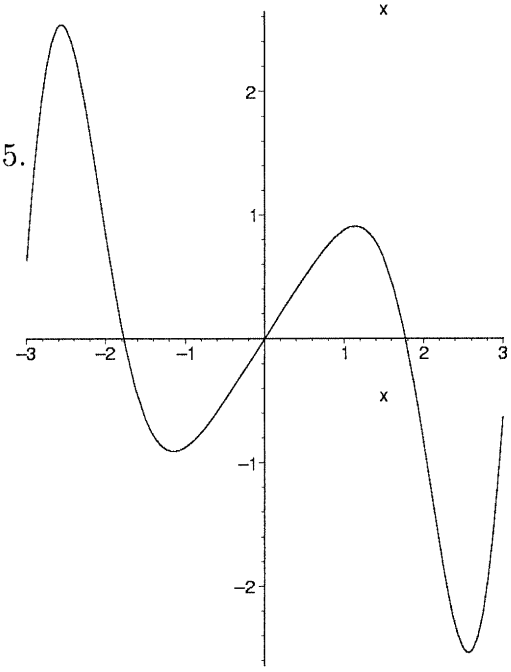
4.


30
25
20
15
10
5
−3
−2
−1
0
1
2
3
x

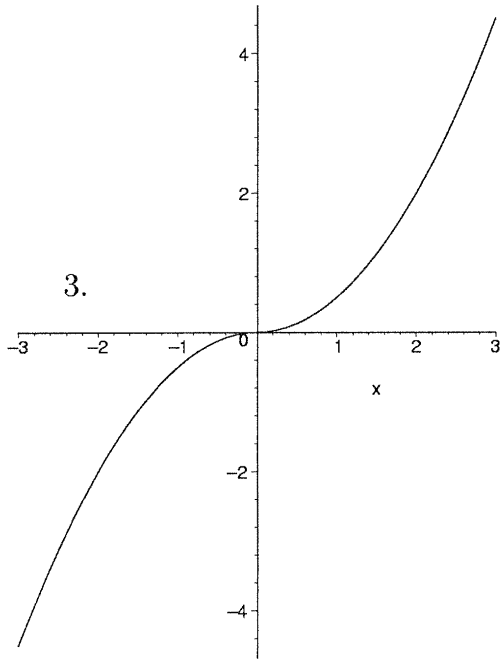
2.


1
0.5
−0.5
−1
−3
−2
−1
0
1
2
3
x

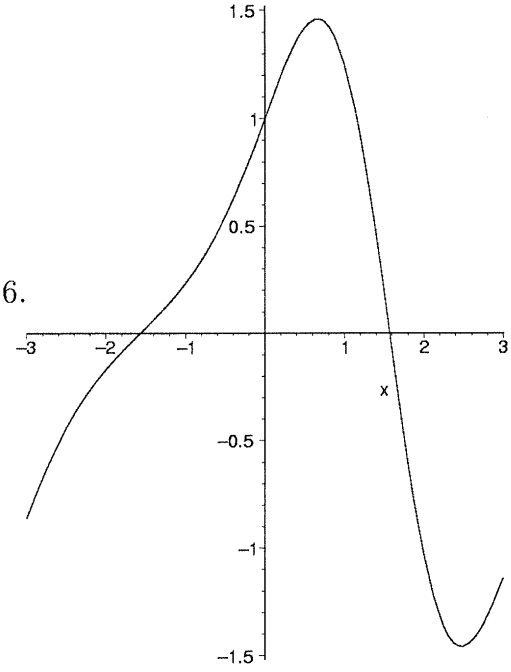
5.


2
1
−1
−2
−3
−2
−1
1
2
3
x

3.

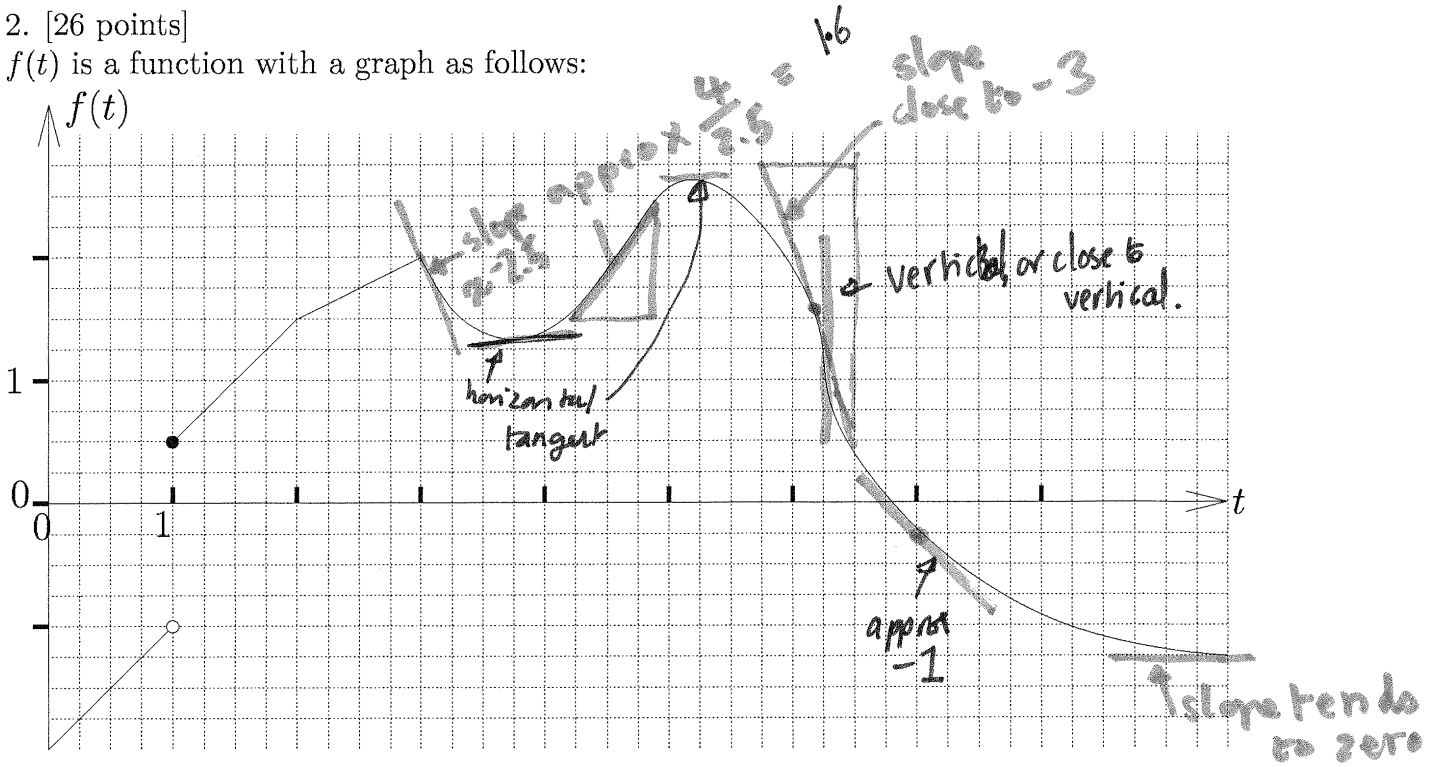

4
2
−2
−4
−3
−2
−1
0
1
2
3
x

6.


1.5
1
0.5
−0.5
−1
−1.5
−3
−2
−1
1
2
3
x

2. [26 points]

$f(t)$ is a function with a graph as follows:



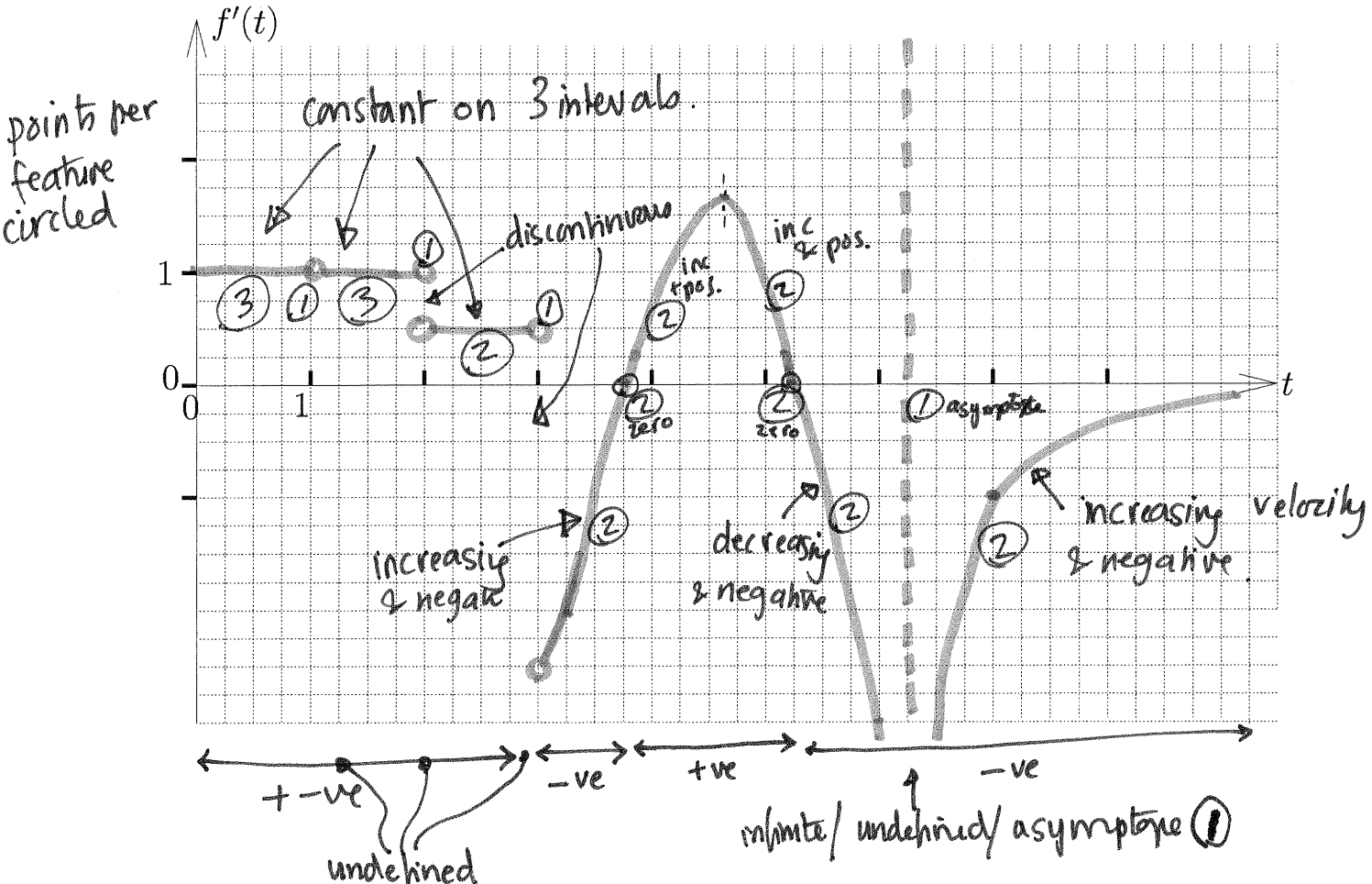
Sketch the graph of $f'(t)$ below. Make sure your graph shows the following features:

i Where $f'(t)$ is zero, positive and negative. ← 12 points

ii Where $f'(t)$ is decreasing or increasing. ← 8 points

iii Where $f'(t)$ is undefined, infinite (by drawing an asymptote), or has a discontinuity. ← 1 each, total 4 points.

iv Exact values of $f'(t)$ when $f'(t)$ is constant on some interval. ← 2 points



3. [20 points] For which values of x does the graph of the following function have horizontal tangent?

$$f(x) = \frac{(1+x)}{(x^2+8)}.$$

$$f'(x) = \frac{(x^2+8)\times 1 - (1+x)2x}{(x^2+8)^2} = \frac{-x^2-2x+8}{(x^2+8)^2} = \frac{-(x+4)(x-2)}{(x^2+8)^2}$$

$f'(x) = 0$ when $x = -4$ or 2

Knowing quotient formula : 5 points
computing it correctly : 5 points
factorization 5 points
final answer : 2½ points for each value.

4. Find the derivatives of the following functions. [5 points each.]
i. $f(x) = x^2e^x$

$$f'(x) = 2xe^x + x^2e^x$$

ii. $f(x) = 4^x$

$$f'(x) = \ln(4)\,4^x$$

iii. $f(x) = e^{(2x^2+3)}$

$f'(x) = 4x e^{(2x^2+3)}$

iv. $f(x) = \dfrac{x^3 - 1}{\sin(x)}$ $\quad f'(x) = \dfrac{\sin(x)\,3x^2 - (x^3-1)\cos(x)}{(\sin(x))^2}$

v. $f(x) = \cos(x^2)$ $\quad f'(x) = -2x \sin(x^2)$

vi. $f(x) = \sqrt{\sin\left(\dfrac{1}{x}\right)}$ $\quad f'(x) = -x^{-2}\cos\left(\frac{1}{x}\right) 1/2 \left(\sin\left(\frac{1}{x}\right)\right)^{-1/2}$

$= \dfrac{-\cos(1/x)}{2x^2\sqrt{\sin(1/x)}}$