Differentiation

Basic functions

polynomial (3.1)	$(x^n) = nx^{n-1}$	
exponential (3.1)	$(e^x)' = e^x$	$(a^x)' = \ln(a)a^x$
trigonometric (3.4)	$(\sin(x))' = \cos(x)$	
	$(\cos(x))' = -\sin(x)$	
	$(\tan(x))' = \sec^2(x)$	
logarithmic (3.8)	$(\ln(x))' = 1/x$	$(\log_a(x))' = 1/(\ln(a)x)$

Special role of e^x (3.1, 3.8)

- e^x is its own derivative
- derivative of inverse, $\ln(x)$ is $1/x$
- slope of tangent at 0 is 1
- $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

Building from basics

addition rule (3.1)	$(f+g)' = f' + g'$
subtraction rule (3.1)	$(f-g)'$=f'-g'
times constant rule (3.1)	$(cf)' = cf'$
product rule (3.2)	$(fg)' = f'g + g'f$
quotient rule (3.2)	$(f/g)' = (gf' - g'f)/g^2$
chain rule (3.5)	$(f \circ g)' = g' \times (f' \circ g)$

Higher derivatives

(3.7) e.g. distance → velocity → acceleration

Applications

Rates of change (3.3)
Related rates (3.10)
Determining errors (3.11)

Meaning of derivative

Limit
definition (2.8)

$f'(a) =$

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

Graph
of derivative (2.9)

draw your own example:

Slope
of Tangent (2.7)

draw your own example:

Techniques

Implicit differentiation (3.6)

need to use chain rule: $\frac{df(y)}{dx} = y'\frac{df(y)}{dy}$
1) use when y not directly in terms of x:
e.g: $y^2 + x^3 = xy$
2) use to find derivative of inverses
e.g.: $y = \sin^{-1}(x)$

Logarithmic differentiation (3.8)

Method: take logs, take derivative, multiply by $f(x)$
1) use when $f(x)$ involves lots of powers
e.g: $f(x) = (x^3+x+1)^7(x+5)^3\sqrt{\sin(x)}(5x+4)^{-2}$
2) use when $f(x) = g(x)^{h(x)}$
e.g.: $f(x) = (x^3+2)^{\cos(x)}$