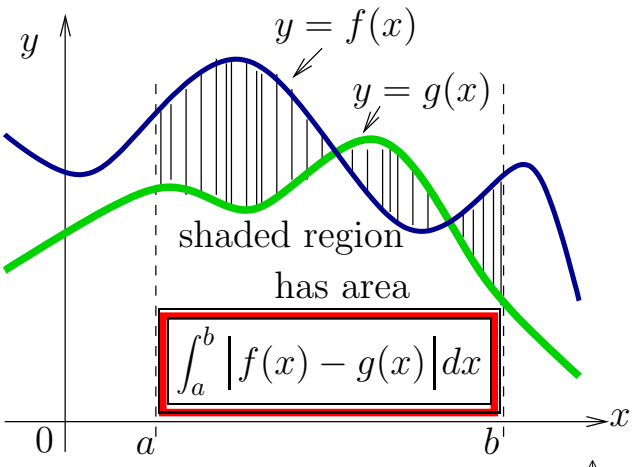
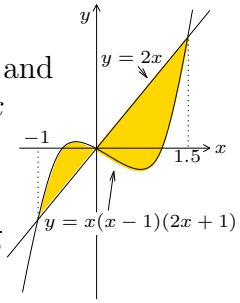
Math 1550, section 6, Spring 2004. Chapter 6 – at a glance

Area between curves, §6.1 (Tuesday April 27)

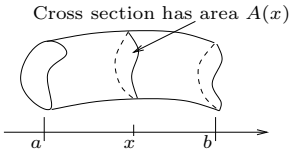


E.g., the finite area between $y = 2x$ and $y = x(2x+1)(x-1) = 2x^3 - x^2 - x$ is $\int_{-1}^{0} [(2x^3 - x^2 - x) - 2x]dx$ $+ \int_0^{1.5} [2x - (2x^3 - x^2 - x)]dx = 2\frac{61}{96}$

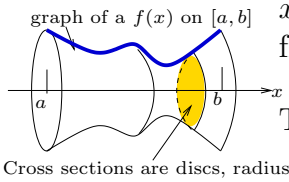


Possible exam questions: like §6.1 ex 1, 2, 7, 13, and quiz/handout. Only involving curves of the form $y =$ polynomial in x.

Volumes, (rotation about x-axis) §6.2 (Monday, Tuesday May 3, 4)



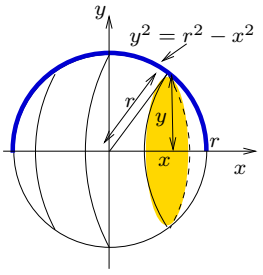
For objects built from **slices**, volume is $\int_a^b A(x)dx$



Rotating a curve about the x-axis creates an object built from **discs** of radius $f(x)$.

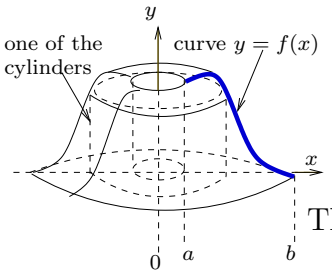
The volume is $\int_a^b \pi(f(x))^2 dx$

E.g., For a sphere radius r, take $y = f(x) = \sqrt{r^2 - x^2}$. The volume is $2\int_0^r \pi(r^2 - x^2)dx =$ $\left[\pi\left(r^2 x - \frac{1}{3}x^3\right)\right]_0^r = 2\pi\frac{2}{3}r^3 = \frac{4}{3}\pi r^3$



Possible exam questions: like §6.2 ex 1, 3, and quiz/handout. Only involving polynomials and rotation about about $x-$axis.

Volumes, (rotation about y-axis) §6.3 (Tuesday, Wednesday May 4, 5)



Rotating a curve about the y-axis creates an object built from **cylindrical shells** of radius x and height $f(x)$.

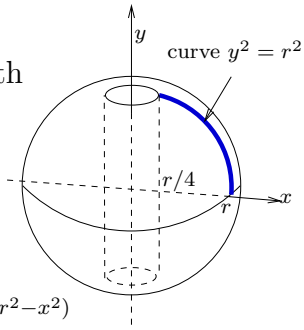
The volume is $\int_a^b 2\pi x f(x)dx$

E.g., a bead (radius r) with a hole through the middle (radius $r/4$) has volume

$2\int_{r/4}^{r} 2\pi x\sqrt{r^2 - x^2}\,dx =$

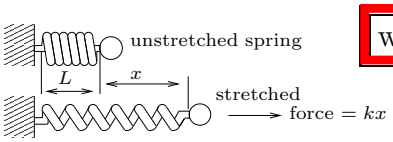
$2\int_{15r^2/16}^{0} -\pi\sqrt{u}\,du$ (set $u = r^2 - x^2$)

$= -2\left[\pi\frac{2}{3}u^{3/2}\right]_{15r^2/16}^{0} = 5\pi\sqrt{15}\,r^3/16 \approx 3.8\,r^3$



Possible exam questions: like §6.3 ex 1, 5, 6, 7 and quiz/handout. Only involving polynomials and rotation about about $y-$axis.

Work, §6.4 (Thursday April 29)



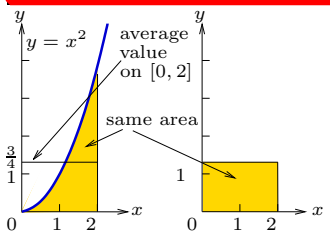
work = force × distance

E.g., work done in stretching a spring from length $L + a$ to $L + b$ is $\int_a^b kx\,dx = \frac{k}{2}(b^2 - a^2)$

Possible exam questions: Only involving springs, like §6.4, example 3

Average value of $f(x)$, §6.5 (Friday April 30)

Average value of $f(x)$ on $[a, b]$ is $f_{\text{ave}} = \frac{1}{b-a}\int_a^b f(x)\,dx$



E.g., average of $f(x) = x^2$ on $[0, 2]$ is $\frac{1}{2}\int_0^2 x^2\,dx =$ $\frac{4}{3} = f(2/\sqrt{3})$

Mean value theorem for integrals: The average of $f(x)$ on $[a, b]$ is attained at some $c \in [a, b]$

Possible exam questions: like §6.5 ex 1–10 (but no trig) and quiz/handout.