

Math1550, section 6. Calculus. FINAL EXAM, Wednesday, May 12 2004,
Locket 134, 5:30—7:30 pm

STUDENT NAME:

This exam has 12 pages, and 9 questions (some questions have more than one part).
There are 10 points per question, except for question 6, which is worth 20 points.

Please put your initials at the top of each page.

Please show your working.

1. [10 points] (Important: read the whole question before starting to sketch!)

In the grid below, sketch a graph with domain $[-4, 4]$ which satisfies:

0. $f(x)$ is continuous on $[-4, -2) \cup (-2, 2) \cup (2, 4]$

i. $\lim_{x \rightarrow -2^-} f(x) = 2$

ii. $\lim_{x \rightarrow -2^+} f(x) = -3$

iii. $f(-2) = -1$

iv. $\lim_{x \rightarrow 2} f(x) = -1$

v. $f(2) = 4$

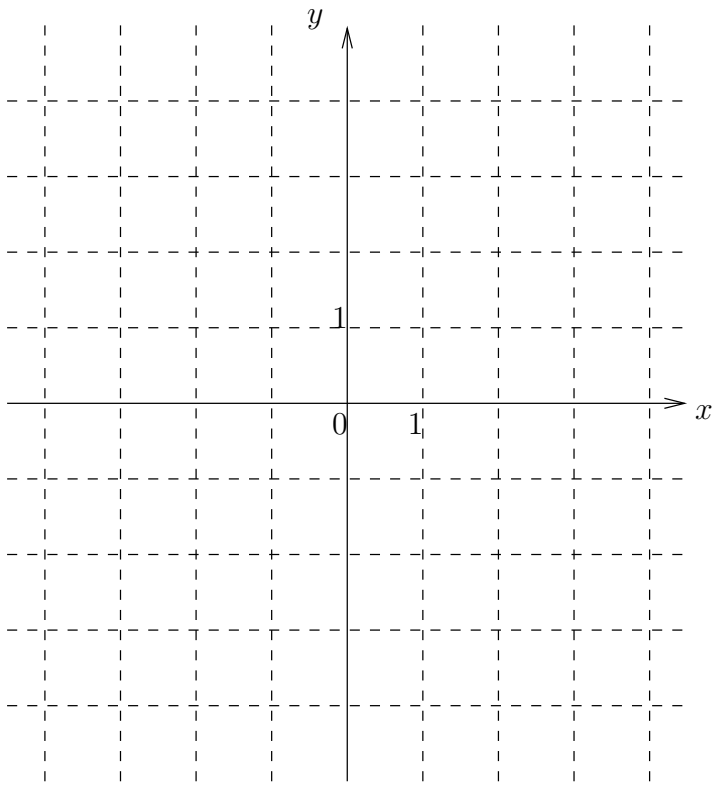
vi. f is increasing on $(-4, -2)$

vii. f is decreasing on $(2, 4)$

viii. f is concave up on $(-2, 0)$

ix. f is concave down on $(0, 2)$

x. f has a local maximum when $x = 1$.



2. [10 points] What is the absolute minimum value of the function

$$f(x) = x^3 - 6x^2 + 9x + 1$$

on the interval $[0, 2]$?

3. [10 points]

i. What is the largest integer less than $\sqrt[5]{40}$?

ii. Starting with your answer from part i, use Newton's method applied to $y = x^5 - 40$ to find a better approximation to a solution to $\sqrt[5]{40}$. (Write your answer as a fraction.)

4. [10 points] Find the derivatives of the following functions.

i. $f(x) = x^7 + \tan(x) - 2^x$

ii. $f(x) = \sin(\cos(x))$

iii. $f(x) = x^{e^x}$

5. [10 points] Use implicit differentiation to find $\frac{dy}{dx}$.

i. $\cos(x) \sin(y) = 1$

ii. $\ln(y^3) = x^2$

6. [20 points] Let $f(x)$ be the function

$$f(x) = \frac{1 + x^3}{1 - x^3}.$$

i. What is the domain of f ? _____

ii. Is f odd, even, or periodic, or has no symmetries? _____

iii. What are the x and y intercepts for $f(x)$?

iv. Does $f(x)$ have vertical asymptotes, and if so, what are they?

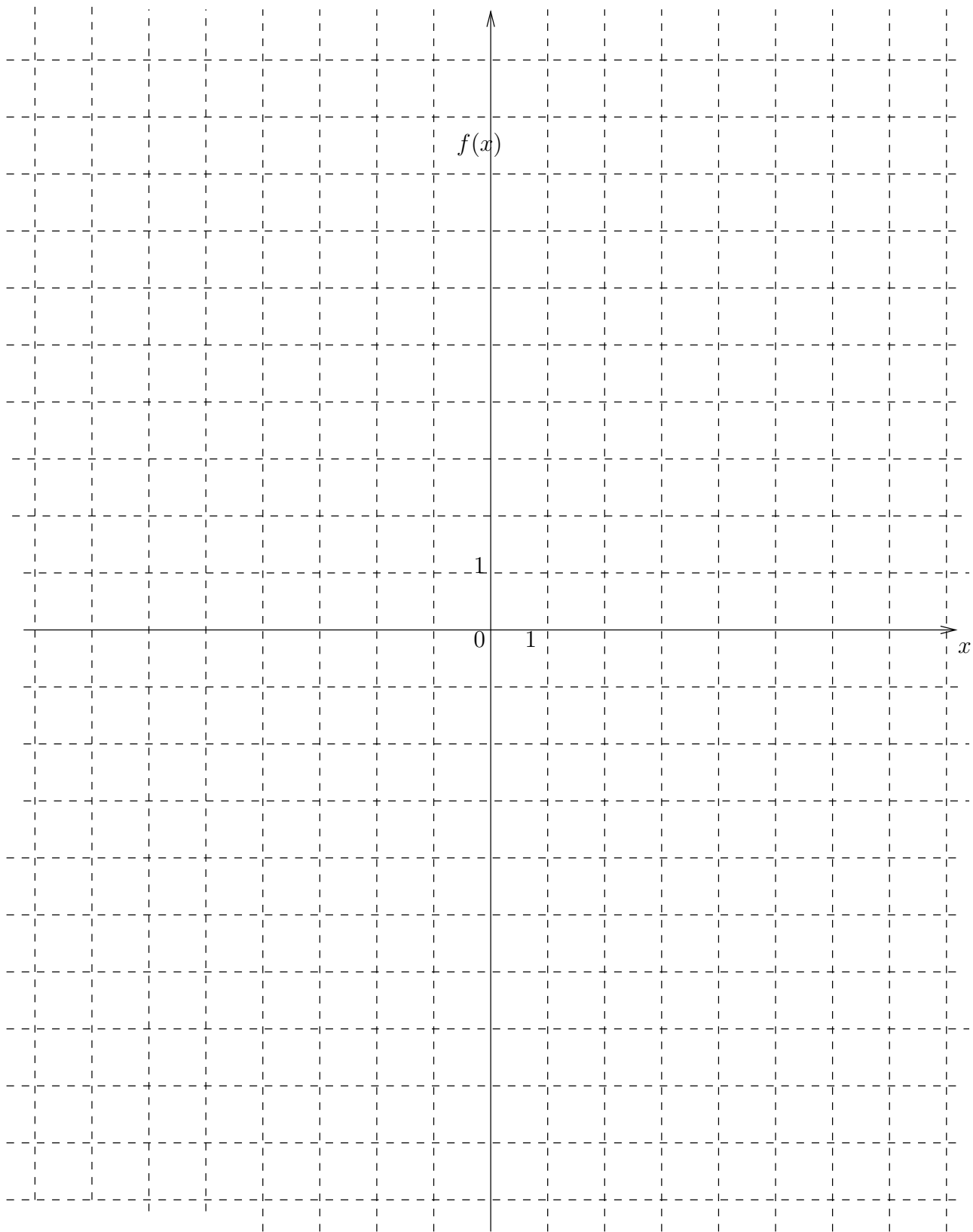
v. Does $f(x)$ have horizontal asymptotes, and if so, what are they? _____

vi. What are the critical numbers of $f(x)$?

What is the value of $f(x)$ at the critical numbers in the domain of $f(x)$?

vii. On which intervals is $f(x)$ increasing? _____

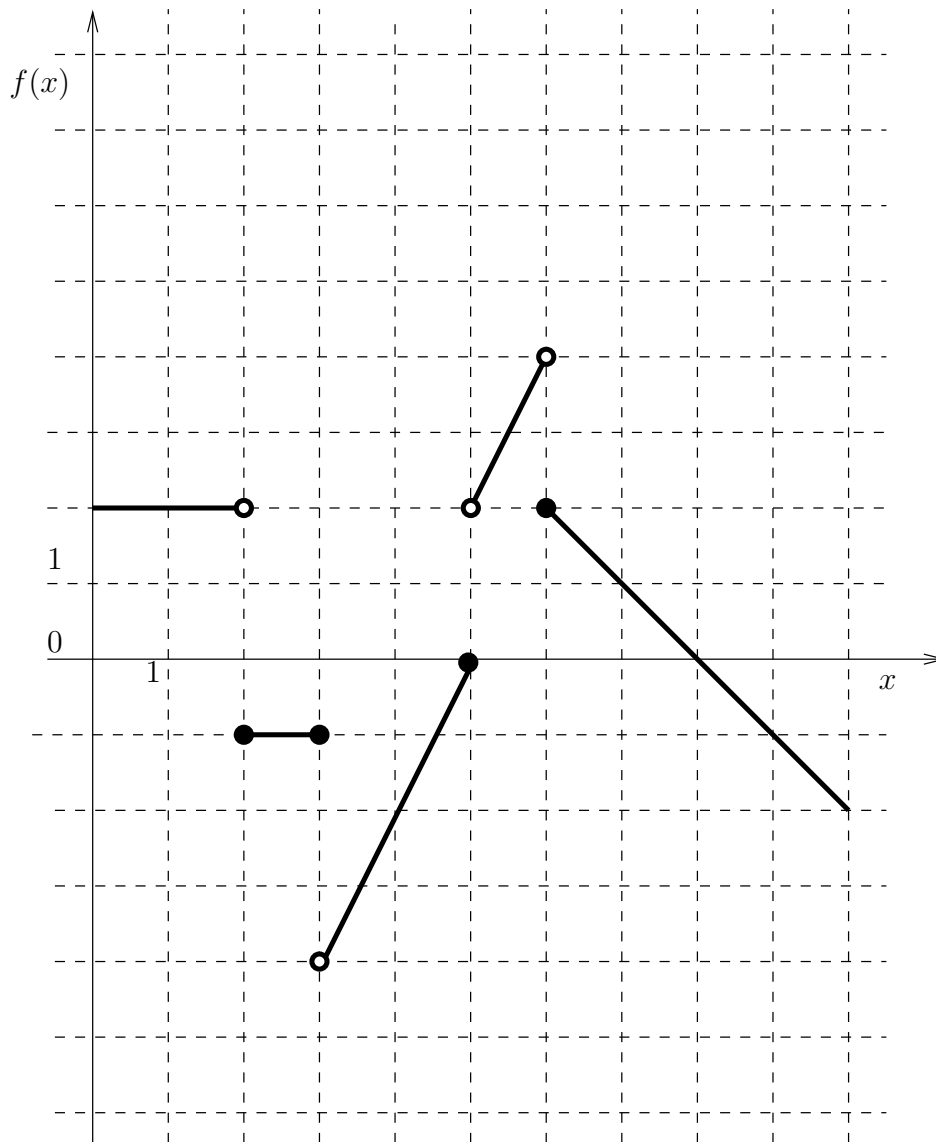
viii. Sketch a graph of $f(x)$, showing the above features.



7. [10 points] Below the graph of a function $f(x)$ is sketched.

i. On the same grid, Sketch a graph of the function

$$g(x) = \int_0^x f(t) dt$$



ii. What is the maximum value of $g(x)$ on $[0, 10]$?

iii. What is the minimum value of $g(x)$ on $[0, 10]$?

iv. Suppose

$$\int_a^x f(t) dt = g(x) - 3$$

What is a possible value of a ?

8. [10 points] Find the following definite and indefinite integrals. Show your working and reasoning. In the indefinite cases, give the most general form.

i. $\int \frac{x^2 + 1}{x} dx.$

ii. $\int_0^2 \frac{x}{x^2 + 1} dx.$

iii. $\int x^2(8 - x^3)^9 dx$

iv. $\int_{-1}^1 x^4 \sin(x^5) dx$

v. $\int_1^1 \sin^9(x) dx$

9. [10 points] What is the volume of the solid obtained by rotating the area between the curve $y = e^x$ and the x axis, from $x = 0$ to $x = 1$, about the x -axis?
(Leave your answer as an expression in terms of e and π .)