1550, §6, Spring 2004. Practice questions for preparation for the final exam:

1. (a) Sketch the curve $y = (x-1)(2x+1)$.
   (b) Sketch the tangent to this curve at $x = 3$, and find the equation of this tangent.
   (c) Draw a secant to this curve, with slope 7, and passing through the point with $x = 3$. What is the equation for this secant? Where else does this secant cross the curve?

2. Find the following limits: i. $\lim_{x\to 0^+} \frac{|x^2-1|}{x-1}$; ii. $\lim_{x\to 0} \sqrt{\frac{\sin(9x)}{\sin(4x)}}$; iii. $\lim_{x\to\infty} \frac{e^{4x}}{e^{5x}+\cos(x)}$; iv. $\lim_{x\to 0} x^{\sin(x)}$.

3. Find an interval of length 1 containing a solution to $2^x = x^3$.

4. Figure A below shows a function $f(x)$, and $f'(x)$, $f''(x)$ and $f'''(x)$ — which is which?

5. Figures $B1, B2, B3, B4$ below show $g(x), g'(x), h(x)$ and $h'(x)$ — which is which?

6. (a) Sketch a curve with: a horizontal asymptote; 1 vertical asymptote; 2 discontinuities; 3 places where the derivative is discontinuous; 1 local maximum; 2 local minimums.
   (b) Sketch the derivative of the function you have just sketched.

7. Find the derivatives of the following functions: i. $\cos(x^2+1)$; ii. $\sin^4(x)$; iii. $\cos(e^x)$; iv. $\tan(x)^e$; v. $\ln(x^4+2)$; vi. $\ln(x)\tan(x)$; vii. $\sin^{-1}(x^2+1)$; viii. $\sqrt[3]{(\sin(x)+x)}$.

8. Find dy/dx in the following cases: i. $x^3 + xy = y^3$; ii. $(x+2y)^{10} = \tan(y-x)$; iii. $1 = xye^y$.

9. What is the maximum area of a rectangle with perimeter 12?

10. If two boats start at the same point, and one travels North at 10km/hr and the other East at 20km/hr, what is their relative speed after 2 hours?

11. Find some $c \in [1,2]$ satisfying the mean value theorem for $f(x) = x^2 - 3x + 5$ on this interval.

12. Find the domain, intercepts, symmetries, asymptotes of i) $f(x) = (x+3)/(x^2-1)$; find also the critical values, intervals of increasing, decreasing, concave up and concave down, for ii) $f(x) = (x-3)/(x^2+1)$. Sketch both curves.

13. Guess a solution to $x^3 = 10$, and use Newton's method to find a better solution from your first guess.

14. Estimate the value of $\int_{1/2}^{3/2} x^{-2}dx$ by dividing $[0.5, 1.5]$ into 4 subintervals, and approximating the area this integral represents by 4 rectangles.

15. Find a function $f(x)$ with $f'(x) = 6x^2 + 4$ and $f(-1) = 0$.

16. Compute the following integrals: i. $\int_{-1}^{1}(3x^5+x^3+x)dx$; ii. $\int_{-1}^{1}(4x^6+x^2+1)dx$; iii. $\int_1^2 \frac{3x^2+2x}{x^3+x^2+1}dx$; iv. $\int_{-\pi}^{\pi} x^2\cos(x^3)dx$; v. $\int_0^2 x^2\sqrt{x^3+1}dx$; vi. $\int_1^2 \frac{\sin(1/x)}{x^2}dx$.

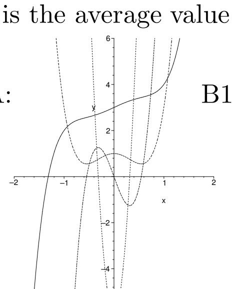
17. Let $g(x) = x$ on $[0,1)$ and $[2,4)$; $g(x) = -x$ on $[1,2)$ and $[4,6)$, and $g(x) = 0$ elsewhere.
    i. Sketch a graph of $g(x)$. ii. Sketch a graph of $g'(x)$. iii. Sketch a graph of $f(x) := \int_0^x g(t)dt$.
    iv. What are the absolute maximum and minimum values of $f(x)$?
    v. Find a value of a such that $\int_a^x g(t)dt = f(x) - 2$.

18. What is the area between the curves $f(x) = 6x^4 + 2$ and $g(x) = 5x^2 + 1$?

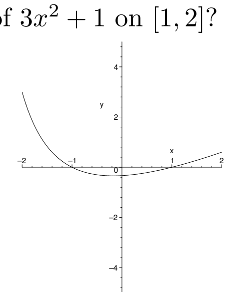
19. What is the volume of the solid obtained by rotating the curve $y = x^2 - 4x + 5$ with $x \in [1,3]$,
    (a) about the x-axis? (b) about the y-axis?

20. What is the average value of $3x^2 + 1$ on $[1,2]$?

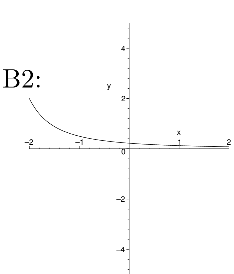
Figure A:



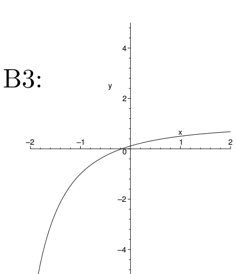
B1:



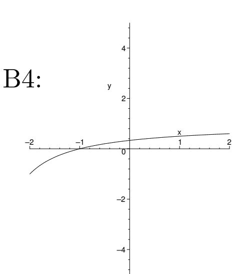
B2:



B3:



B4:



Things to know for the final exam:

Chapter 2: Tangents and Limits

- Tangents and secants: how to find equations for them
- Limits; how the slope of a tangent is defined by a limit; how to use limits to compute asymptotes.

Chapter 3: Differentiation

- Derivatives of polynomials, exponentials, logarithm, trig functions
- Rules: Product, quotient and chain rule
- Implicit differentiation
- Higher derivatives
- Recognising, and sketching the derivative of a function given a graph of the function

Chapter 4:

- Optimisation—maximum and minimum values of functions
- Mean value theorem (average value of rate of change)
- L'hopital's rule (especially for computing asymptotes)
- Graph sketching—learn the list of features to look for (domain, …concavity, etc) and how to compute them
- Newton's method of finding approximate solutions to equations
- Antiderivatives

Chapter 5:

- Definite integral and indefinite integral—how to compute using antiderivatives
- How to estimate integrals by rectangular strips
- Antiderivatives of polynomials, some trig and exponential functions
- Substitution method of integration
- Fundamental theorem of calculus (i.e., that integration and differentiation are opposites)
- Sketching the integral function of a function

Chapter 6:

- Area between two curves
- Volumes
- Average value of a function