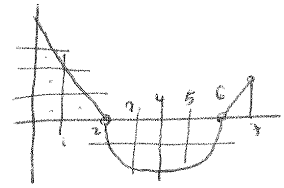
Homework 5

5.2. 34 :

$g(x)$:



a) $\int_0^2 g(x)\,dx = 4$

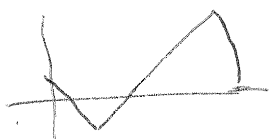
b) $\int_2^6 g(x)\,dx = -$ area of $1/2$ circle, radius 2

$= -1/2 \times \pi 2^2 = -2\pi$

$\approx -2 \times 3.14 = -6.28$

c) $\int_0^7 g(x)\,dx = 4 - 2\pi + 1/2 = 4.5 - 6.28 \approx -1.78$

5.3. ex 2

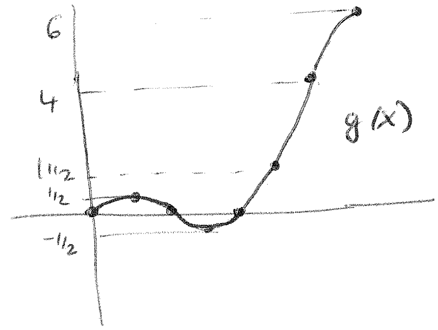


$g(x) = \int_0^x f(t)\,dt$

a) ~~$g(x)$~~

x	$g(x)$	
0	~~$1/2$~~	0
1	~~$1/2 - 1/2 = 0$~~	$1/2$
2	~~$0 - 1/2 = -1/2$~~	0
3	$-1/2$	$-1/2$
4	$-1/2 + 1/2 = 0$	
5	$0 + 1\tfrac{1}{2} = 1\tfrac{1}{2}$	
6	$1\tfrac{1}{2} + 2\tfrac{1}{2} = 4$	

d)



b) $g(7) = g(6) + \int_6^7 f(t)\,dt$

$\int_6^7 f(t)\,dt$ looks like about area 2,

so, $g(7) \approx g(6) + 2 = 6$

c) g has max value at $x = 7$,

min value at $x = 3$

5.4, ex 8

$\int x(1+2x^4)\,dx = \int (x + 2x^5)\,dx = 1/2\, x^2 + \frac{2}{6} x^6 + C$

$= 1/2\, x^2 + 1/3\, x^6 + C$

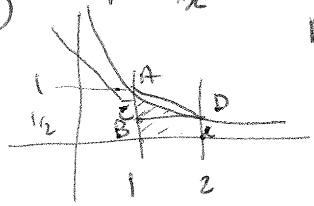
5.5 ex 4

$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$, $u = \sqrt{x}$ $\Rightarrow du = 1/2\, x^{-1/2}\,dx$ $\Rightarrow \frac{dx}{\sqrt{x}} = 2\,du$

$= \int \sin(u)\, 2\,du = -\cos(u)2 + C = -2\cos(\sqrt{x}) + C$

5.6 a)

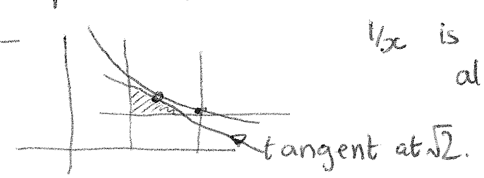
$f(x) = 1/x$



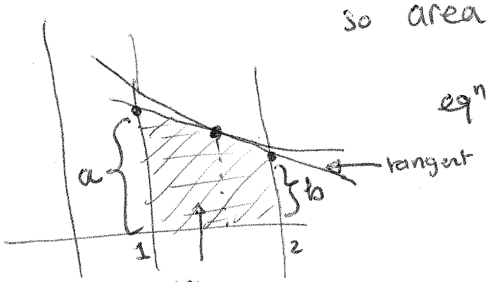
line AD has slope $\frac{1/2 - 1}{2-1} = \frac{-1/2}{1} = -1/2$

mean value theorem says $f'(c) = -1/2$ for some $c \in [1,2]$.

$f'(x) = -1/x^2$ so at slope $-1/2$, $-1/c^2 = -1/2 \Rightarrow c^2 = 2 \Rightarrow c = \sqrt{2}$



$1/x$ is concave down, since $f'(x) = -1/x^2$ is always -ve. So $1/x$ is above tangent so area under tangent at $\sqrt{2}$ is less than $\int_1^2 \frac{1}{x}\,dx = \ln(2)$



area under tangent $= \frac{a+b}{2}$

eqn of tangent is

$$\frac{y - 1/\sqrt{2}}{x - \sqrt{2}} = -1/2$$

$$y - 1/\sqrt{2} = -1/2\, x + \sqrt{2}/2 = -1/2\, x + 1/\sqrt{2}$$

$$\boxed{y = -1/2\, x + 2/\sqrt{2} = -1/2\, x + \sqrt{2}}$$

so, if $x = 1$, point on tangent at $x = 1$ is $(1, -1/2 + \sqrt{2})$
$x = 2$ - - - - - - - $x = 2$ is $(2, -1 + \sqrt{2})$

so area under tangent from 1 to 2 is $\frac{-1/2 + \sqrt{2} + -1 + \sqrt{2}}{2} = \frac{-3/2 + 2\sqrt{2}}{2}$

$= \sqrt{2} - 3/4$

so $\ln(2) > \sqrt{2} - 3/4$

need to check that $\sqrt{2} - 3/4 > 0.66$

$\Leftrightarrow \sqrt{2} > 0.66 + 0.75 = 1.41$

$\Leftrightarrow 2 > (1.41)^2 = (1 + 0.41)^2$

$= 1 + 2 \times 0.41 + (0.41)^2$ $\quad (0.4 + 0.01)^2$

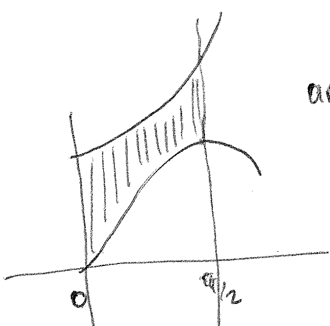
$= 1 + 0.81 + 0.16 + 0.008 + 0.0001$

$= 1.9681$

which is true, so $\ln(2) > 0.66$.

6.1 ex 6

$y = \sin(x)$
$y = e^x$
$x = 0$
$x = \pi/2$



area $= \int_0^{\pi/2} (e^x - \sin(x))\,dx$

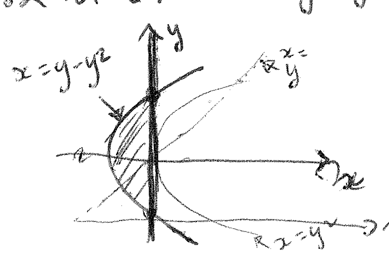
$= \left[e^x + \cos(x) \right]_0^{\pi/2} = e^{\pi/2} + \cos(\pi/2) - e^0 - \cos(0)$

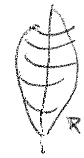
$= e^{\pi/2} + 0 - 1 - 1$

$= e^{\pi/2} - 2$

6.2 ex 6. area betw $x = y - y^2$, $x = 0$ about y-axis

the curves intersect at $y = 0$ & 1





object made of discs

Volume $= \int_0^1 \pi (y - y^2)^2\,dy = \int_0^1 \pi (y^2 - 2y^3 + y^4)\,dy$

$= \pi \left[\frac{1}{3}y^3 - \frac{1}{2}y^4 + \frac{1}{5}y^5 \right]_0^1$

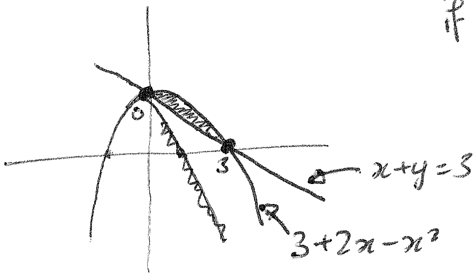
$= \pi (1/3 - 1/2 + 1/5) = \pi \frac{10 - 15 + 6}{30} = \frac{\pi}{30}$

6.3 ex 6.

$y = 3 - x$

$y = 3 + 2x - x^2, \quad x + y = 3$ about y-axis

$= (3-x)(1+x)$



if curves cross,

$\cancel{3} - x = \cancel{3} + 2x - x^2$

$3x = x^2 \Rightarrow x$ or 0 or 3

$$\text{Volume} = \int_0^3 2\pi x\left(\cancel{3} + 2x - x^2 - (\cancel{3} - x)\right)dx$$

$$= \int_0^3 2\pi x(3x - x^2)\,dx$$

$$= \int_0^3 2\pi(3x^2 - x^3)\,dx$$

$$= \left[2\pi\left(\cancel{3}x^3/\cancel{3} - \tfrac{1}{4}x^4\right)\right]_0^3 = 2\pi\left(27 - \tfrac{81}{4}\right) = \frac{27\pi}{2}$$

$$= \cancel{2\pi(8 - 16/4) = 2\pi(8-4) = 8\pi.}$$

6.4 ex 2 how much work to lift 60kg to 2m?

work = force × distance

$F = mg = 60 \times 9.8$

(gravity)

$d = 2$

$$\text{work} = 60 \times 9.8 \times 2$$
$$= 120 \times 9.8$$
$$= 120 \times (10 - 0.2)$$
$$= 1200 - 24 = 1176\ \text{N}$$

6.5 ex 8

$h(r) = 3/(1+r)^2$ on $[1, 6]$

$$\text{average} = \frac{1}{6-1}\int_1^6 \frac{3}{(1+r)^2}\,dr$$

let $u = 1 + r$

then $du = dr$

& if $r = 1$, $u = 2$

$r = 6 \quad u = 7$

$$= \frac{1}{5}\int_2^7 \frac{3}{u^2}\,du$$

$$= \frac{1}{5}\left[\frac{3u^{-1}}{-1}\right]_2^7 = \frac{1}{5}\left(3\left(-\tfrac{1}{7} - -\tfrac{1}{2}\right)\right)$$

$$= \tfrac{3}{5}\left(\tfrac{1}{2} - \tfrac{1}{7}\right) = \tfrac{3}{5}\,\tfrac{5}{14} = \tfrac{3}{14}.$$