

STUDENT NAME:

Calculus 1550, section 6. Wednesday March 3, 2004. Tenth quiz

1. [3 points] What is the slope of the tangent to

$$y = e^x \sqrt{x^3} = e^x x^{3/2}$$

at the point $(1, e)$?

$$\begin{aligned} y' &= e^x (x^{3/2})' + (e^x)' \cdot x^{3/2} \\ &= e^x \cdot \frac{3}{2} x^{1/2} + e^x x^{3/2} \\ &= e^x x^{1/2} (3/2 + x) \end{aligned}$$

at $x = 1$, $y' = e^1 \cdot 1 \cdot (3/2 + 1) = \frac{5e}{2}$, so slope of tangent at $(1, e)$ is

$$\boxed{\frac{5e}{2}}$$

Remark: can use chain rule (not necessary):
 $(\sqrt{x^3})' = \frac{1}{2}(x^3)^{-1/2} \cdot 3x^2$
deriv of $y^{1/2}$ deriv of x^3
 $= \frac{3}{2} x^{-3/2} \cdot x^2 = \frac{3}{2} x^{2-3/2}$
 $= \frac{3}{2} x^{1/2}$

2. [3 points] Find the derivative of the following function

$$f(x) = e^x x^3 + \frac{x^2}{x^2 + 1}$$

$$\begin{aligned} f'(x) &= e^x (x^3)' + (e^x)' x^3 + \frac{(x^2+1)(x^2)' - (x^2+1)' x^2}{(x^2+1)^2} \\ &= e^x 3x^2 + e^x x^3 + \frac{(x^2+1)2x - 2x x^2}{(x^2+1)^2} \\ &= e^x x^2 (3+x) + \frac{2x^3 + 2x - 2x^3}{(x^2+1)^2} \\ &= \boxed{e^x x^2 (3+x) + \frac{2x}{(x^2+1)^2}} \end{aligned}$$

3. [4 points] Where does the graph of the following function have horizontal tangents?

$$y = (x^2 - 3)e^x$$

$$\begin{aligned} y' &= (x^2 - 3)(e^x)' + (x^2 - 3)' e^x \\ &= (x^2 - 3)e^x + 2xe^x = e^x(x^2 + 2x - 3) \\ &= e^x(x+3)(x-1) \end{aligned}$$

the tangent is horizontal when $y' = 0$

ie, when

$$\boxed{x = -3 \quad \text{or} \quad x = 1}$$

(because e^x is never zero)