STUDENT NAME:

Calculus 1550, section 6. Thursday, March 11, 2004. Thirteenth quiz.

Find $\frac{dy}{dx}$ by using implicit differentiation, in each of the following cases. Your answer may be written as a function involving both x and y.

1.[3 points] $xy = x^2 + y^2$

$$(xy)' = (x^2 + y^2)'$$

$$x'y + y'x = (x^2)' + (y^2)'$$

$$1 \times y + y'x = 2x(x)' + 2y\,y'$$

$$y + y'x = 2x + 2y\,y'$$

$$y'(x - 2y) = 2x - y \longrightarrow y' = \frac{2x - y}{x - 2y}$$

2.[3 points] $\frac{1}{y} = e^x$

$$\left(\frac{1}{y}\right)' = (e^x)'$$

$$y'(-y^{-2}) = e^x$$

$$\frac{y'}{-y^2} = e^x$$

$$\Rightarrow y' = -y^2 e^x$$

(alternatively: $\frac{1}{y} = e^x$

$\Rightarrow y = 1/e^x = e^{-x}$

$\Rightarrow y' = (-x)' \times e^{-x}$

$= -e^{-x}$)

3.[4 points] $\cos(y^2) = xe^y$

$$(\cos(y^2))' = (xe^y)'$$

$$(y^2)'(-\sin(y^2)) = (x)'e^y + x(e^y)'$$

$$-2y\,y'\sin(y^2) = e^y + x\,y'e^y$$

$$y'(-2y\sin(y^2) - xe^y) = e^y$$

$$\Rightarrow y' = \frac{-e^y}{2y\sin(y^2) + xe^y}$$