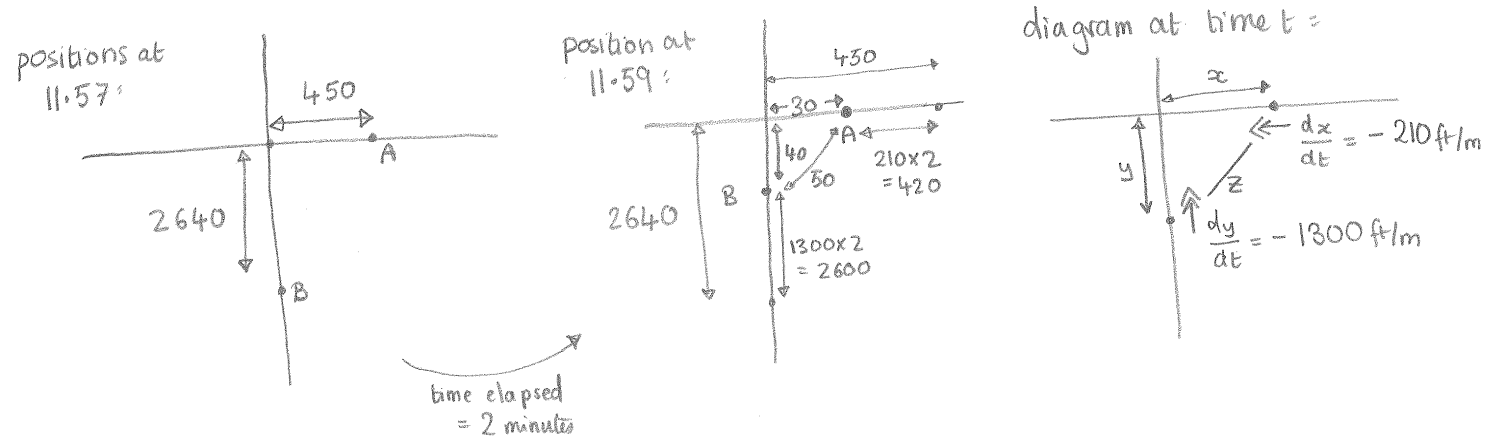


STUDENT NAME:

Calculus 1550, section 6. Thursday, March 18, 2004. Fourteenth quiz

Ann and Ben have arranged to meet at noon at the intersection of South Campus Drive and Highland Road, two roads which meet at right angles. At 11:57am, Ann is on South Campus Drive, 450 ft from the meeting point. She walks towards the meeting point at 210 ft/min. At the same time, Ben is on Highland Road, 1/2 a mile (=2640 ft) away from the meeting point. He runs towards the meeting point at 1300 ft/min. Assuming they continue at constant speeds, at what speed are they approaching each other at 11:59am?

Points are given for 1) Drawing and labelling a diagram, making your notation clear, and stating values the variables and their derivatives [3 points]; 2) making it clear what relation you find which needs to be differentiated [2.5 points]; 3) differentiating correctly [2.5 points]; 4) obtaining the final answer [2 points].



let x = distance of Ann from meeting pt
 y = " " Ben " "
 z = distance from Ann to Ben

Relation: $x^2 + y^2 = z^2$

So $\frac{d(x^2)}{dt} + \frac{d(y^2)}{dt} = \frac{d(z^2)}{dt} \Rightarrow$

$2x x' + 2y y' = 2z z'$

(prime means diff. with respect to t)
 $t = \text{time}$

So $z' = \frac{xx' + yy'}{z}$

When $t = 11:59$, $x = 30$, $y = 40$, $z = 50$ ($= \sqrt{30^2 + 40^2}$)
 $x' = -210$ $y' = -1300$

So $-z' = \frac{30 \times 210 + 40 \times 1300}{50} = \frac{3 \times 2 \times 5 \times 21 \times 10 + 40 \times 13 \times 100}{50}$

$= \frac{3 \times 2 \times 21 \times 50 + 40 \times 13 \times 2 \times 50}{50}$

$= 6 \times 21 + 40 \times 26 = 126 + 1040 = 1166 \text{ ft/min}$

So at 11:59, they are approaching each other at 1166 ft/min